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A RECURSIVE ESTIMATION ALGORITHM FOR IDENTIFICATION OF DYNAMIC SYSTEMS

TONEY R. PERKINS
JAMES F. LEATHRUM

APRIL 1974

U.S. ARMY MATERIEL SYSTEMS ANALYSIS AGENCY
Aberdeen Proving Ground, Maryland

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Toney R. Perkins
James F. Leathrum*

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*Associate Professor, Computer Science Department, University of Delaware. Dr. Leathrum is Consultant to AMSAA.

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JFLeathrum/TRPerkins/cep
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ABSTRACT

This report deals with the identification of unknown parameters in dynamic systems. In modeling a physical system, the problem of identifying the dynamics of a system are often encountered. The developed algorithm provides a tool to model all or parts of a dynamic system using input-output data sets from a real system. The methodology and techniques of this algorithm are based upon linear recursive estimation theory. The theoretical foundation and the pragmatics of utilizing the ensemble data to estimate the unknown parameters are discussed at length in the development of the algorithm.

As an application of the algorithm, experimental data from a man-in-the-loop simulation is used to estimate the parameters of a single axis model of the gunner. The tracking response of the gunner model compare favorably with data obtained from the simulation. The differences in tracking responses are attributed to not including human randomness in the model.

FOREWORD

The effort described in this report was expended in support of the Vulcan Air Defense System (VADS) effectiveness task.

The authors wish to acknowledge P. E. Corcoran, H. H. Burke, C. L. Cairns, Jr., and P. J. Randall for their support and contributions to the successful completion of the task. Special acknowledgement is extended to P. J. Randall whose programming and hybrid computer expertise made it possible to accomplish the objectives of the task.

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A RECURSIVE ESTIMATION ALGORITHM FOR IDENTIFICATION OF DYNAMIC SYSTEMS

1. INTRODUCTION

This report is a discussion and documentation of an algorithm for identification of dynamic systems. The proposed algorithm is closely related to other sequential estimation algorithms proposed by Kalman,[2] Mayne,[3] Deutch,[1] and others over the past decade. The most significant and unique feature of the algorithm described here is the extent to which computational efficiency and accuracy are achieved and assured. Particular attention is given to handling of ill-conditioned computations, and non-stationary models.

The algorithm described here was first implemented on a small computer for a real time application and subsequently has continued to be refined for small, highly interactive computers. The interactive features of the algorithm have proven to be invaluable in all stages of an estimation project. The analyst may interact early in an application to tune the free parameters of the algorithm and throughout production of estimates to control computing time, convergence of the estimates, and computer output.

The viewpoint taken in developing and listing the algorithm described here was that the data upon which the estimates are based arrive one at a time in a discrete unedited form. The capacity to store old data and the computer time are both very limited resources. The objective of the algorithm is to compute a new model from each new data set utilizing a compacted form of the data history. The extent to which the history is utilized is controlled by the algorithm in such a way as to minimize the prediction error for the model.

The remainder of this report is a discussion of the theoretical foundations of the algorithm and the pragmatics of data collection and model validation.

2. THEORY

2.1 Statistical Model.

The basic model of the random process to be observed is

$$\underline{Y} = \underline{X}\,\underline{\beta} + \underline{V} \quad (N \times 1) \qquad 2.1.1$$

Where $\underline{Y}$ is the vector of observations, $\underline{X}$ $(N \times P)$ is the matrix of controlled variables, $\underline{\beta}$ $(P \times 1)$ is the vector of parameters to be estimated, and $\underline{V}$ $(N \times 1)$ is the vector of observation errors. The observation errors are presumed to have well known first and second moments defined by

$$E(\underline{V}) = \underline{0} \qquad 2.1.2$$

$$E(\underline{V}\,\underline{V}^T) = \underline{\psi} \qquad 2.1.3$$

We wish to compute an estimate, $\hat{\underline{\beta}}$, of $\underline{\beta}$ such that

$$\hat{\underline{\beta}} = \underline{K}\,\underline{Y} \qquad 2.1.4$$

and $$E(\hat{\underline{\beta}}) = \underline{\beta} \text{ (unbiased)}$$

This implies directly that

$$\underline{K}\,\underline{X} - \underline{I} = \underline{0} \text{ for all } \underline{\beta} \qquad 2.1.5$$

The estimation problem may be formulated as a minimum variance problem where $\underline{K}$ must be found such that

$$\mathrm{Var}(\hat{\underline{\beta}}) = \mathrm{Tr}\, E\left[(\hat{\underline{\beta}}-\underline{\beta})(\hat{\underline{\beta}}-\underline{\beta})^T\right] \qquad 2.1.6$$

in minimized, subject to Equation 2.1.5. The minimization of Equation 2.1.6 leads directly to

$$\underline{K} = (\underline{X}^T\underline{\psi}^{-1}\underline{X})^{-1}\underline{X}^T\underline{\psi}^{-1} \qquad 2.1.7$$

2.2 Model of the Observation Errors.

In the previous section it was presumed that the covariance of the observation errors was known. We will now develop a statistical model of the observation errors which produces a sequence of covariance matrices, $\underline{\psi}_i$, where i is conceptually an index on time. The model will represent an acknowledgement that observation errors increase with the

age of the observation.

$$\underline{\Psi}_{i+1} = \left[\begin{array}{c|c} \frac{\sigma^2}{\alpha} & \\ \hline & \left(\frac{1}{1-\alpha}\right)\underline{\Psi}_i \end{array}\right] \qquad 2.2.1$$

In this model variances occur at the value, $\frac{\sigma^2}{\alpha}$ and increase by a factor $\frac{1}{1-\alpha}$ in each time increment. The variances are thus increasing exponentially with age such that diagonal elements, t units of time in age, are equal to $(1-\alpha)^{-t}\,\frac{\sigma^2}{\alpha}$.

It is convenient in the development which follows to represent the error variance model as

$$\underline{\Psi}_{i+1} = \sigma^2\, \underline{Q}_{i+1} \qquad 2.2.2$$

where

$$\underline{Q}_{i+1} = \left[\begin{array}{c|c} \frac{1}{\alpha} & \\ \hline & \left(\frac{1}{1-\alpha}\right)\underline{Q}_i \end{array}\right] \qquad 2.2.3$$

$$Q_o = 1/\alpha$$

Using Equation 2.2.2, the estimation gain matrix $\underline{K}$, reduces to

$$\underline{K} = (\underline{X}_{i+1}\underline{Q}_{i+1}^{-1}\,\underline{X}_{i+1})^{-1}\underline{X}_{i+1}\,\underline{Q}_{i+1}^{-1} \qquad 2.2.4$$

Where $\underline{X}_{i+1}$ is the same as $\underline{X}$ of Equation 2.1.7. The subscript of $\underline{X}_{i+1}$ affords a means of writing a recursive definition

$$\underline{X}_{i+1} = \left[\begin{array}{c} \underline{x}_{i+1}^T \\ \hline \underline{X}_i \end{array}\right] \qquad 2.2.5$$

where $\underline{x}_{i+1}^T$ is the vector of most recent values of the controlled variables. Likewise the observations may be defined recursively as

$$\underline{Y}_{i+1} = \left[\begin{array}{c} \underline{y}_{i+1} \\ \hline \underline{Y}_i \end{array}\right] \qquad 2.2.6$$

Using Equations 2.2.4, 2.2.5, and 2.2.6 the estimation equation

(Equation 2.1.4) now becomes

$$\hat{\underline{\beta}}_{i+1} = (\underline{X}_{i+1}^{T}\ \underline{Q}_{i+1}^{-1}\ \underline{X}_{i+1})^{-1}\ \underline{X}_{i+1}^{T}\ \underline{Q}_{i+1}^{-1}\ \underline{Y}_{i+1} \qquad 2.2.7$$

Further use of the stepwise recursion for $\hat{\beta}$ leads to

$$(\alpha\ \underline{x}_{i+1}\ \underline{x}_{i+1}^{T} + (1-\alpha)\underline{X}_{i}^{T}\underline{Q}_{i}^{-1}\underline{X}_{i})\Delta_{i}\hat{\underline{\beta}} = \alpha\underline{x}_{i+1}(y_{i+1}-\underline{x}_{i+1}^{T}\hat{\underline{\beta}}_{i}) \qquad 2.2.8$$

where

$$\Delta_{i}\hat{\underline{\beta}} = \hat{\underline{\beta}}_{i+1} - \hat{\underline{\beta}}_{i}$$

Equation 2.2.8 represents the starting point for the development of the recursive estimation algorithm. The assumption involved in the use of this equation are

1) Observation model (Equation 2.1.1)
2) Linear Estimation (Equation 2.1.4)
3) Unbiased Estimation (Equation 2.1.5)
4) Minimum Variance Estimation (Equation 2.1.6)
5) Known Error Moments (Equation 2.1.2 and 2.1.3)
6) Exponential Error Variances (Equation 2.2.1)

Of particular note here is the fact that no assumptions are made regarding distribution functions. Such assumptions may be delayed until the estimates, $\hat{\underline{\beta}}$ and their variances must be interpreted statistically.

2.3 Statistical Interpretation.

It was shown in the previous development that the recursive estimation scheme (Equation 2.2.8) produces an unbiased, minimum variance estimate of the parameters, $\hat{\underline{\beta}}$. The estimator, $\hat{\underline{\beta}}$, is itself a random variable thus a statistical measure of its closeness to the true parameters, $\hat{\underline{\beta}}$, is needed. Such a measure is the covariance matrix, $\underline{P}$.

$$\underline{P}_{i+1} \equiv E\left[(\hat{\underline{\beta}}_{i+1} - \underline{\beta}_{i+1})(\hat{\underline{\beta}}_{i+1} - \underline{\beta}_{i+1})^{T}\right] \qquad 2.3.1$$

Using Equation 2.2.7 and Equation 2.1.1 the covariance matrix may be written as

$$\underline{P}_{i+1} = \sigma^{2}(\underline{X}_{i+1}^{T}\ \underline{Q}_{i+1}^{-1}\ \underline{X}_{i+1})^{-1} \qquad 2.3.2$$

Once again a recursive formulation may be employed

$$\underline{P}_{i+1} = \sigma^2 \left(\alpha \underline{x}_{i+1} \underline{x}_{i+1}^T + \sigma^2(1-\alpha) \underline{P}_i^{-1}\right)^{-1} \tag{2.3.3}$$

or equivalently by the matrix inversion lemma[1]

$$\underline{P}_{i+1} = \frac{1}{1-\alpha}[\underline{P}_i - \alpha \underline{P}_i \underline{x}_{i+1} (\alpha \underline{x}_{i+1}^T \underline{P}_i \underline{x}_{i+1} + (1-\alpha)\sigma^2)^{-1} \underline{x}_{i+1}^T \underline{P}_i] \tag{2.3.4}$$

The preceding result illustrates how the covariance matrix may be propagated forward in time without the inverse indicated in Equation 2.3.3. In fact only scalar inversion is required in Equation 2.3.4.

In the development of Equations 2.3.3 and 2.3.4 it was presumed that the necessary matrix inverses always exist. In particular the existence of $(\underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1})^{-1}$ was never questioned. It is quite possible in the progression of this matrix that it becomes singular for all practical purposes. For example, if the data became constant (i.e. $\underline{x}_{i+1} = \underline{x}_i$ for all i), then the recallable history will look like a single sample. The rank of the matrix, $\underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1}$, is then one.

In the case of singular or very poorly conditioned solutions of Equation 2.2.8, we may wish to employ a generalized inverse which would minimize some suitable norm of $\Delta_i \hat{\underline{\beta}}$. Thus with an incomplete data set we would hold the parameter estimates, $\hat{\underline{\beta}}$, as constant as possible. The covariance matrix in this case may be found by first rearranging Equation 2.2.7 and substituting Equation 2.1.1.

$$(\underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1})\hat{\underline{\beta}}_{i+1} = \underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1} \underline{\beta}_{i+1} + \underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{V}_{i+1} \tag{2.3.5}$$

Now employing a generalized inverse leads to

$$\hat{\underline{\beta}}_{i+1} - \underline{\beta}_{i+1} = (\underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1})^{\mathsf{T}} \underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{V}_{i+1} \tag{2.3.6}$$

where, T, indicates generalized inverse. The covariance matrix follows from its definition

$$\underline{P}_{i+1} = \sigma^2 (\underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1})^{\mathsf{T}} \underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1} [(\underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1})^{\mathsf{T}}]^T = \sigma^2 (\underline{X}_{i+1}^T \underline{Q}_{i+1}^{-1} \underline{X}_{i+1})^{\mathsf{T}} \tag{2.3.7}$$

The interpretation of the covariance matrix, $\underline{P}_{i+1}$, is closely connected to the interpretation of the error covariance matrix, $\underline{\Psi}_{i+1}$. If the latter represents the second moment of a Gaussian distribution, then $\underline{P}_{i+1}$ may be interpreted as the second moment of a Gaussian distribution of the estimate $\hat{\underline{\beta}}$. The covariance matrix, $\underline{P}_{i+1}$, defines the variance ellipsoids analogous to the familiar σ, 2σ, 3σ, etc. limits of a Gaussian distribution in one dimension.

2.4 Computational Considerations.

From a computational point of view, the recursive estimation scheme, Equation 2.2.8 is a linear algebraic equation of the form

$$\underline{A}_{i+1}\,\underline{Z}_{i+1} = \underline{B}_{i+1} \tag{2.4.1}$$

where
$$\underline{A}_{i+1} \equiv (\alpha\,\underline{x}_{i+1}\underline{x}_{i+1}^{T} + (1-\alpha)\,\underline{X}_{i}^{T}\,\underline{Q}_{i}^{-1}\,\underline{X}_{i}) \tag{2.4.2}$$

$$\underline{Z}_{i+1} \equiv \Delta_{i}\hat{\underline{\beta}} \tag{2.4.3}$$

$$\underline{B}_{i+1} \equiv \alpha\,\underline{x}_{i+1}(y_{i+1} - \underline{x}_{i+1}^{T}\,\hat{\underline{\beta}}_{i}) \tag{2.4.4}$$

The matrix, $\underline{A}$, is symmetric and is of uncertain rank. Both $\underline{A}$ and $\underline{B}$ are functions of the parameter, α, which hereafter will be viewed as a parameter which may be reset at each **time**. Thus any information generated in computing the estimator, $\hat{\underline{\beta}}_{i+1}$, may also be used to determine the covariance of the observation errors. Thus the statistics of the observation errors may be subjected to adaptation. Since it is intended here that $\hat{\underline{\beta}}_{i+1}$ be used as a predictor in the interval (i+1, i+2), the most appropriate value of α is that which minimizes the single step prediction error. If such an α can be found for $\hat{\underline{\beta}}_{i}$, it will simply be used to compute $\hat{\underline{\beta}}_{i+1}$. The single step prediction error, E_r, is defined by

$$E_r = (y_{i+1} - \underline{x}_{i+1}^{T}\,\hat{\underline{\beta}}_{i}) \tag{2.4.5}$$

The minimum of the squared prediction error, σ^2, will be approximated by a one step quadient search over the parameter, α

where
$$\alpha_{i+1} = \alpha_i - \varepsilon \left.\frac{ds}{d\alpha}\right|_{\alpha_i} \quad 2.4.6$$

$$\left.\frac{ds}{d\alpha}\right|_{\alpha_i} \equiv 2E_r \underline{x}_{i+1}^T \left.\frac{d\hat{\beta}_i}{d\alpha}\right|_{\alpha_i} \quad 2.4.7$$

$$A_i \frac{d\hat{\beta}_i}{d\alpha} = (1-\alpha)^{-1} \underline{x}_i \, (y_i - \underline{x}_i^T \hat{\underline{\beta}}_i) \quad 2.4.8$$

Note that in this adjustment of α the only matrix inversion required is the same one completed in the computation of $\hat{\underline{\beta}}$. By such a scheme a new value of α may be found with very little computation over and above that required to compute $\hat{\underline{\beta}}$.

In computing the estimator, $\hat{\underline{\beta}}_{i+1}$, it must be recognized that $\underline{A}_{i+1}$, may become nearly singular. In such degenerate cases the computation of the estimator is very poorly conditioned. In order to have a measure of approach to singularity, a recursive bound for the eigenvalues of $\underline{A}_{i+1}$ is computed. Using Equation 2.4.2 and the Rayleigh Quotient it can be shown that

$$(\lambda_1)_{\underline{A}_{i+1}} \geq (1-\alpha)(\lambda_1)_{\underline{A}_i} \quad 2.4.9$$

$$(\lambda_n)_{\underline{A}_{i+1}} \leq (1-\alpha)(\lambda_n)_{\underline{A}_i} + \alpha \underline{x}_{i+1}^T \underline{x}_{i+1} \quad 2.4.10$$

where λ_1 and λ_n are the smallest and largest eigenvalues respectively. ($(\lambda_i)_A$ should be read "the ith eigenvalue of A"). By computing these bounds, the condition of $\underline{A}_{i+1}$ can be approximated and used to trigger a generalized inversion whenever necessary.

2.5 Identification of Discrete Systems

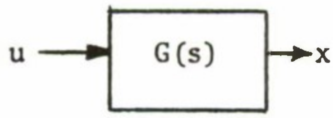
The recursive estimation technique developed in the previous section will now be used as a basis for identification of dynamical systems. As an example suppose the system to be identified is modelled as a linear discrete system of the form,

$$\underline{X}_{i+1} = \underline{\phi}\,(i, i+1)\, \underline{X}_i + \underline{B}\, \underline{U}_i \quad 2.5.1$$

where $\underline{X}_i \equiv$ state at time i

$\underline{\phi}(i, i+1) \equiv$ state transition matrix from i to i+1.

As an example consider a single input, single output system.



In such a system the state vector will consist of

$$\underline{X}_i = \begin{bmatrix} u_i \\ E^{-1} x_i \\ E^{-2} x_i \end{bmatrix} \qquad 2.5.2$$

and the input forcing vector may be

$$\underline{U}_i = \begin{bmatrix} \underline{u}_i \\ E^{-1} u_i \\ E^{-2} u_i \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \qquad 2.5.3$$

where $E^n x_i \equiv x_{i+n} = Z^{+n} x_i$

($E^n \equiv E(E(E(---(\cdot)(---)$; n applications of E)
n times

In this example suppose the Z-transfer function was

$$G(Z) = \frac{eZ^3 + fZ^2}{Z^4 - aZ^3 - bZ^2 - cZ - d} \qquad 2.5.4$$

The state variable representation of such a system is

$$\underline{X}_{i+1} = \begin{bmatrix} a & b & c & d \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underline{X}_i + \begin{bmatrix} e & f \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \underline{U}_i \qquad 2.5.5$$

$$\text{where} \quad X_i \equiv \begin{bmatrix} x_i \\ x_{i-1} \\ x_{i-2} \\ x_{i-3} \end{bmatrix} ; \; \underline{U}_i \equiv \begin{bmatrix} u_i \\ u_{i-1} \end{bmatrix}$$

The objective of an identification algorithm for such a situation is the estimation of the values of a, b, c, d, e, and f. The recursive estimation algorithm described in the previous section is directly applicable after the following transformations are made

Discrete System Variables	Estimation Variables
x_i	y_i
$[x_{i-1}, x_{i-2}, x_{i-3}, x_{i-4}, u_{i-1}, u_{i-2}]$	$\underline{X}_i^T$
$[a,b,c,d,e,f]$	$\underline{\beta}^T$

The viewpoint employed here is that the most recent output observation is subject to error, ε. There is an underlying assumption that the other variables, $\underline{X}_i$, are error free. To the extent that such an assumption is not valid, these errors may be thought of as included in ε by the following

$$\varepsilon_i = \text{Error in } x_i + \beta_{i-1}^T \cdot \; [\text{Error in } \underline{X}_{i-1}] \qquad 2.5.6$$

A more formal representation of these errors is not tractable at the present time, either by conventional regression analysis or by filtering theory.

2.6 Identification of Continuous Systems.

In contrast to the previous section we will now develop a formulation of a continuous system which will permit its identification by the proposed algorithm. Most of the information suggested in the

literature require the use of derivatives of state variables as observations.[1,4] This is to be avoided if possible because of the excessive errors involved in observing or approximating these derivatives.

The system to be considered here is of the form,

$$\dot{\underline{X}} = \underline{A}\,\underline{X} + \underline{B}\,\underline{U} \quad (N\times 1) \qquad 2.6.1$$

Where $\underline{A}$ and $\underline{B}$ are the parameters (perhaps time dependent) which must be estimated. The analytic solution of this equation over the time interval is

$$\underline{X}_{i+1} = \underline{\phi}\,\underline{X}_i + \underline{R}\,\underline{U}_i$$

where

$$\underline{\phi} = \exp\,(\underline{A}\Delta\tau) \qquad 2.6.2$$

$$\underline{R} = (\underline{\phi} - \underline{I})\,\underline{A}^{-1}\underline{B}$$

and, where for the moment, $\underline{A}$ and $\underline{B}$ are treated as constants. Now Equation (2.6.2) is of the same form as Equation 2.5.1, but the state variables are not interrelated as in Equations 2.5.2 and 2.5.3. Here the state variables have been carried over from the continuous form, Equation (2.6.1).

The identification problem has now been cast in the form of a problem of estimating $\underline{\phi}$ and $\underline{R}$. Using the algorithm described in the section on multivariate observations, the estimates are obtained by first transforming

Equation (2.6.2)	Estimation Equations
$\underline{\phi}$ and $\underline{R}$	$\underline{\beta}$
$\underline{X}_i$ and $\underline{U}_i$	$\underline{X}_i$
$\underline{X}_{i+1}$	$\underline{Y}_i$

The estimates, $\hat{\underline{\beta}}$, represent an identification of the discrete form of the original system. The conversion back to the continuous form is most readily accomplished by

1) Noting that $\underline{\phi}$ and $\underline{A}$ have the same eigenvalues, first diagonalize $\underline{\phi}$

$$\underline{\phi} = \underline{S}\ \underline{D}_{\phi}\underline{S}^{-1} = e^{\underline{A}\tau} = \underline{S}e^{\underline{D}_A \Delta\tau}\underline{S}^{-1} \qquad 2.6.3$$

$$\underline{D}_A = \frac{1}{\Delta\tau} \text{Ln } \underline{D}_{\phi} \qquad 2.6.4$$

$$\underline{A} = \underline{S}\ \underline{D}_A\underline{S} \qquad 2.6.5$$

where D_{ϕ} and D_A are diagonalizations of ϕ and A respectively and the above computations may involve complex arithmetic.

2) From $\underline{A}$, ϕ, and $\underline{R}$; $\underline{B}$ may be recovered

$$\underline{B} = \underline{A}\ (\underline{\phi} - \underline{I})^{-1}\underline{R} \qquad 2.6.6$$

2.7 Multivariate Observations.

The derivations of the previous sections are readily extended to include more than one observation at each time step. Beginning with the model of the random process

$$\underline{y}_i = \underline{x}_i\ \underline{\beta}_i + \underline{V}_i$$

$$E(\underline{V}_i) = 0 \qquad 2.7.1$$

$$E(\underline{V}_i\underline{V}_i^T) = R_i$$

We wish to estimate, $\hat{\underline{\beta}}_i$, such that

$$\hat{\underline{\beta}}_i = \sum_{j=o}^{i} k_j\underline{y}_j \qquad 2.7.2$$

$$E(\hat{\underline{\beta}}_i) = \underline{\beta}_i$$

var $(\hat{\underline{\beta}}_i)$ is minimized.

The observation errors are readily modeled as

$$\underline{R}_i = \frac{1}{\alpha}\underline{r} \qquad 2.7.3$$

$$\underline{R}_{i-t} = \frac{1}{(1-\alpha)^t \alpha} \underline{r} \qquad 2.7.4$$

The estimates are obtained in a form similar to the Kalman Filter formulation

$$\hat{\underline{\beta}}_{i+1} = \hat{\underline{\beta}}_i + \underline{P}_i \underline{x}_i^T R_i^{-1} (y_i - x_i \hat{\beta}_{i-1}) \qquad 2.7.5$$

$$\underline{P}_i = (\alpha \underline{x}_i^T \underline{r}^{-1} \underline{x}_i + (1-\alpha)\underline{P}_{i-1}^{-1})^{-1} \qquad 2.7.6$$

or in the special case of

$$\underline{r} = \sigma^2 \underline{I}, \qquad 2.7.7$$

it follows that

$$\hat{\underline{\beta}}_{i+1} = \hat{\underline{\beta}}_i + \alpha \underline{Q}_i \underline{x}_i^T (y_i - \hat{\underline{\beta}}_{i-1}) \qquad 2.7.8$$

$$\underline{Q}_i = (\alpha \underline{x}_i^T \underline{x}_i + (1-\alpha)\underline{Q}_{i-1}^{-1})^{-1}$$

where $\underline{Q}_i \equiv \frac{1}{\sigma^2} \underline{P}_i$

All the eigenvalues of $\underline{Q}_i^{-1}$ lie in the range

$$(1-\alpha)(\lambda_1)_{\underline{Q}_{i-1}^{-1}} \leq (\lambda_j)_{\underline{Q}_i^{-1}} \leq (1-\alpha)(\lambda_n)_{\underline{Q}_{i+1}^{-1}} + \alpha(\lambda_n)_{\underline{x}_i^T \underline{x}_i} \qquad 2.7.9$$

where $(\lambda_j)_Q$ should be read "the $j\underline{th}$ eigenvalue of Q" **and** more conservatively

$$(1-\alpha)(\lambda_1)_{\underline{Q}_{i-1}^{-1}} \leq (\lambda_j)_{\underline{Q}_i^{-1}} \leq (1-\alpha)(\lambda_n)_{\underline{Q}_{i-1}^{-1}} + \alpha \operatorname{Tr}(\underline{x}_i^T \underline{x}_i) \qquad 2.7.10$$

2.8 Single Input - Single Output Models.

Much of the analysis of the previous section is simplified, and the computations improved if one is dealing with a single input-single output model.

Consider for instance

$$\ddot{x} + a\dot{x} + bx = cU(t) \qquad 2.8.1$$

The derivatives may be approximated as follows

$$\ddot{x}_i \approx \frac{\Delta^2 x_i}{\Delta t^2} + 0\ (\Delta t^2)$$

$$\dot{x}_i = (\Delta x_i + \Delta x_{i-1})/(2\ \Delta t) + 0\ (\Delta t^2)$$

When these approximations are used in the original equation 2.8.1, the result is a finite difference equation.

$$\alpha_1 \Delta^2 x_i + \alpha_2(\Delta x_i + \Delta x_{i-1}) + \alpha_3 x_i = cU_i + 0(\Delta t^2) \qquad 2.8.2$$

or

$$(\alpha_1+\alpha_2)\Delta x_i + (\alpha_2-\alpha_1)\Delta x_{i-1} + \alpha_3 x_i = cU_i + 0(\Delta t^2) \qquad 2.8.3$$

Noting that

$$(\alpha_1 + \alpha_2) = \frac{2+\alpha\Delta t}{2}\ \Delta t^{-2}$$

$$\alpha_3 \equiv b$$

Equation 2.8.3 may be converted to

$$\Delta x_i = -\frac{\alpha 3}{\alpha_1+\alpha_2} x_i + \frac{(\alpha_1-\alpha_2)}{(\alpha_1+\alpha_2)} \Delta x_{i-1} + \frac{c}{\alpha_1+\alpha_2} U_i + 0\ (\Delta t^4) \qquad 2.8.4$$

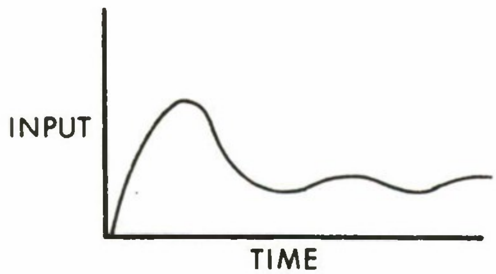
where any approximation in U_i must be good to within $0(\Delta t^2)$. The identification problem is now one of estimating the coefficients of 2.8.4 where the following conversion is made

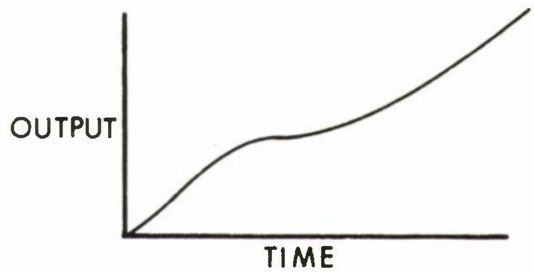
Discrete System Variables	Estimation Variables
Δx_i	y_i
$[x_i,\ \Delta x_{i-1},\ U_i]$	$\underline{x}_i^T$
$\left[\frac{\alpha_3}{\alpha_1+\alpha_2}\ ;\ \frac{\alpha_1-\alpha_2}{\alpha_1+\alpha_2}\ ;\ \frac{c}{\alpha_1+\alpha_2}\right]$	$\hat{\underline{\beta}}^T$

3. DATA HANDLING ASPECTS OF ESTIMATION

3.1 Discussion.

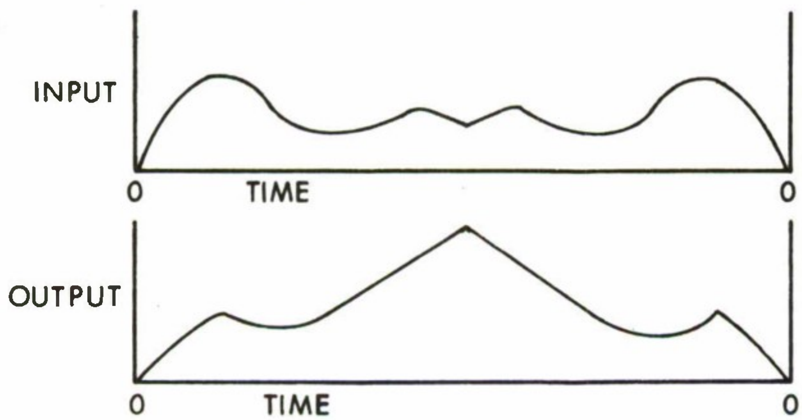
In any actual estimation task there are always a variety of problems which arise because of the form of the data. Where one may wish to have a continuous record of the input-output behavior of the system, in fact it may be a finite record, discretized without regard to the requirements of an estimation algorithm. Many of these problems were actually encountered in the application discussed in the next section. There the data was available in finite records where each record represented one pass of a target for a man-machine system. An example of the input and output data as a function of time is shown below.





There were a great many of the data records as shown above. No one record contained sufficient information for estimation purposes.

In order to obtain a smooth, extended data record, each individual record was repeated in a reverse direction, as illustrated below.



The change to a new data record was always done at time = 0. Thus the multiple data records were concatenated to form one large record without discontinuities. With data in the form illustrated above, the recursive estimation algorithm could be expected to provide an updated model for each sample in the data.

3.2 Converging to a Universal Model.

The model obtained from a recursive estimation algorithm will change from step-to-step in the estimation. As new information becomes available, the estimator will alter the model to minimize the estimation and prediction errors. No matter how accurate such a model is, it may be of limited value if it fails to condense the data to a smaller more fundamental set.

At the cost of somewhat greater prediction errors the estimated model may be forced to converge to a single "universal model" by removing α from Equation 2.2.8 which leads to

$$(\underline{x}_{i+1}\underline{x}_{i+1}^T + \underline{X}_i^T \underline{Q}_i^{-1} \underline{X}_i)\, \Delta_i\hat{\underline{\beta}} = \underline{x}_{i+1}\,(y_{i+1} - \underline{x}_{i+1}^T\hat{\underline{\beta}}) \qquad 3.2.1$$

The variance, $\underline{P}_{i+1}$, tends to zero thus forcing the convergence to the estimates.

3.3 Preliminary Model Validation.

A number of controls are available during estimation to provide measures of performance of the estimated model. At each step of the algorithm discussed in the previous section, the residual error,

$$y_i - \underline{x}_i\,\hat{\underline{\beta}}_i \qquad 3.3.1$$

and the single step prediction error

$$y_{i+1} - \underline{x}_{i+1}\,\hat{\underline{\beta}}_i \qquad 3.3.2$$

are available. Likewise, the covariance matrix, $\underline{P}_i$, is also readily available, but perhaps difficult to use without the statistics of the observation errors being specified. The correlation matrix based upon $\underline{P}_i$ is quite useful in determining the extent to which the model parameters represent the same thing.

Once a model is expected to be valid for a particular data record, that model may be used to reconstruct all the other data records. The reconstructed record may be compared with the actual record on the basis of residual error and prediction error.

It is emphasized that the validation discussed here are preliminary. It means little to have a model reconstruct the data whence it came. This preliminary validation must be viewed as a necessary but not sufficient step in the validation process. The ultimate validation must be based upon the model's ability to "stand along" in place of the actual process. Stability constraints may be required to insure that the dynamic model is a valid replica of the real thing.

4. APPLICATION OF IDENTIFICATION ALGORITHMS.

4.1 Discussion.

The methodology developed in the previous sections is readily applied to the very real problem of understanding the behavior of man in a tracking task. (See Appendix B for description of the system being controlled by man) As a reasonable cause-and-effect model of man we take

$$\ddot{\delta}+\alpha_1\dot{\delta}+\alpha_2\delta = \alpha_3\ (\theta(t-\tau) + \alpha_4\dot{\theta}(t-\tau) + \alpha_5\ddot{\theta}(t-\tau)) \qquad 4.1.1$$

where $\delta \equiv$ Manual output, perturbation from nominal

$\theta \equiv$ Tracking error

$\tau \equiv$ Delay time

Using the methods of Section 2.8, this model is readily discretized to become

$$\Delta\delta_{i-1} = b_1\ \delta_{i-1}+b_2\Delta\delta_{i-2} + b_4\Delta\theta_{i-1-T}+b_4\Delta\theta_{i-2-T}+b_5\Delta\theta_{i-3-T}+0(\Delta t^4) \qquad 4.1.2$$

where $T \equiv \tau\ \ \Delta t$

The model parameters, b_1, b_2,---- b_5, may be used to recover the original parameters, α_1, α_2---- α_5. One would expect both sets of parameters to be time varying and thus require a robost estimator to determine their values. Little will be gained, however, if the estimated parameters are all permitted to vary with time. The resulting model will have more time functions than the original data and as such will not represent a condensation of the data. Such a condensation of the data is critical to establishing a fundamental understanding of the behavior of man.

Although it is acknowledged that man is adaptive, and thus time varying in his description, the search for a universal, constant parameter, man is worth pursuing. The resulting model, if it is valid, would represent a system independent, fundamental understanding of the behavior of man.

4.2 <u>The Search for a Universal Man Model.</u>

The methods of Section 2.4 and 2.8 were adopted to the search for a universal man based upon the model formulation of Equation 4.1.2. Performance data in the form of ($\delta\theta$) pairs were used to estimate the parameters b_1, b_2,----b_5. The original estimates which were time varying were obtained using a fading memory estimator. After the prediction errors settled out to values on the order of 0.05 degrees manual rotational output, the fading memory was removed and the parameters allowed to seek universal values. The resulting model developed a somewhat degraded performance to the level of 0.1 degrees of rotational output error. The universal dimensionless coefficients turned out to be

$$b_1 = -0.0103$$

$$b_2 = 0.272$$

$$b_3 = 0.0368$$

$$b_4 = 8.39 \times 10^{-4} \times 17.453$$

$$b_5 = 0.0135$$

with a time delay of 0.15 sec associated with the tracking error input.

When converted to the continuous form, Equation 4.1.1, the coefficients become

$$\begin{aligned}
\alpha_1 &= 22.9 \text{ sec}^{-1} \\
\alpha_2 &= 6.49 \text{ sec}^{-2} \\
\alpha_3 &= 404. \text{ sec}^{-2} \\
\alpha_4 &= .0672 \text{ sec} \\
\alpha_5 &= .763 \times 10^{-3} \text{ sec}^{-2}
\end{aligned} \tag{4.2.2}$$

4.3 Validation of the Universal Man.

As indicated in the previous section, the universal man was able to reproduce the data whence it was created to within 0.1 degrees of rotational output position. It soon became apparent that such a validation was not good enough. The stability constraints were not included in the estimation algorithm, and as a result the continuous model of man was incapable of handling the control task in a complete system simulation.

Further analysis of the data used to force man (i.e. $\theta(t)$) indicated that the frequency components of $\theta(t)$ were all of lower frequency than natural frequency of man.[5] This suggested that the neuro-muscular terms of the estimated man were spurious (i.e. b_1 and b_2 were probably not estimated correctly).

At this point the estimated man model becomes a point of departure for an orderly search for the correct neuro-muscular response of man.

Previous studies indicate that the eigenvalues of man are limited to at most -10 sec^{-1}.[6] Taking one eigenvalue at -10 sec^{-1}, a second eigenvalue on the order of -580 sec^{-1} gave the most reasonable overall response. On this basis the continuous representation

of man becomes

$$\alpha_1 = +590 \text{ sec}^{-1}$$

$$\alpha_2 = 5800. \text{ sec}^{-2}$$

$$\alpha_3 = 4040. \text{ sec}^{-2}$$

$$\alpha_4 = .0672 \text{ sec}^{-2}$$

$$\alpha_5 = .763 \times 10^{-3} \text{ sec}^{-2}$$

The model is limited by the fact that it is a localized model which in no way models man's behavior in pursuit. The response to track error is a compensatory response or nulling response. In more mathematical terms it is acknowledged that the man model obtained is a perturbation model representing the motion of man with respect to some nominal motion. This nominal motion of man was important in the validation of the man model and was approximated by

$$\delta_{Nominal} = K \dot{A}_T \tag{4.3.2}$$

where $\dot{A}_T$ $\equiv$ Angular rate of the target.

K $\equiv$ Constant on the order of 1.0.

4.4 <u>Summary of the Validation Results.</u>

The man model represented by equations 4.3.1 and 4.3.2 was used in a number of different tracking situations. These results are plotted and shown in the Appendix C. The purpose of this section is to provide a comparison of man's actual performance and the predicted performance on the basis of extremes. One of the most important performance criteria to be applied in judging the performance of a model is the ability to limit the extreme values of track error. The performance of the overall system is closely related to man's ability to limit the extremes of track error. Three distinctly different

phases of performance are available for comparison purposes:

(1) Acquisition phase

(2) Firing phase

(3) Crossover phase

Table 4.4.1 shows the performance of the universal man model (Equations 4.3.1 and 4.3.2 with $K = 1.1$) and the actual man during these phases of tracking.

TABLE 4.4.1

COMPARISON OF MAN MODEL WITH ACTUAL MAN**

TARGET IDENTIFICATION	TARGET* PARAMETERS	MAXIMUM TRACKING ERRORS, MILLIRADIANS ACQUISITION PHASE MODEL	ACQUISITION PHASE MAN	FIRING PHASE MODEL	FIRING PHASE MAN	CROSSOVER PHASE MODEL	CROSSOVER PHASE MAN
MMP1	A=400m V=231.6 m/sec X=450m	6.2	10.5	3.5	3.0	9.4	9
MMP3G2	A=800m V=231.6 m/sec X=450m	6.7	13.6	4.9	5.3	9.4	13
MMP5G2	A=200m V=128.6 m/sec X=450m	3.4	5	2.2	2.5	3.7	3.6
MMP6G1	A=400m V=180 m/sec X=250m	2.7	10.9	3.8	6.7	24.2	20

*A ≡ Altitude, V ≡ Velocity, X ≡ Ground Distance at crossover

**Man Model is Equation 4.3.1 and 4.3.2 with $K = 1.1$

4.5 Approximations to the Man Model.

In the final man model, Equations 4.3.1 and 4.3.2, some of the parameters are of little significance in describing the behavior of man. In particular, the smallest eigenvalue, $-580\ \text{sec}^{-1}$, is of little significance since its effect can only be observed at very high frequencies. Likewise, the $\ddot{\theta}$ term has been found experimentally to have very little effect on the performance. Thus we might consider the following model:

$$\dot{\delta} + \alpha_1 \delta = \alpha_3 \theta(\tau - \tau) + \alpha_4 \dot{\theta}(\tau - \tau) \qquad 4.5.1$$

$$\delta_{Nominal} = K \dot{A}_1$$

where

$$\begin{aligned} \alpha_1 &= 10\ \text{sec}^{-1} \\ \alpha_3 &= 6.97\ \text{sec}^{-1} \\ \alpha_4 &= .0672\ \text{sec} \\ K &= 1.0\ \text{sec} \end{aligned} \qquad 4.5.2$$

A system block diagram of this model is shown in Figure 4.5.1.

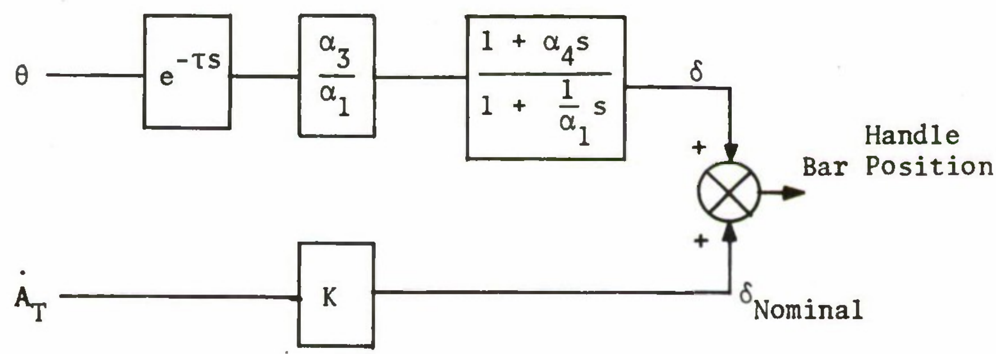


FIGURE 4.5.1 APPROXIMATED MAN MODEL

REFERENCES

1. Deutsch, R., Estimation Theory, Prentice-Hall (1965).

2. Kalman, R., A New Approach to Linear Filtering and Prediction Problems, J. Basic Eng., Trans ASME, 82D, (1960).

3. Leathrum, J. F., On-Line Recursive Estimation, Proc Joint Automatic Control, (Conf). (1969).

4. Mayne, D. Q., Optimal Non-Stationary Estimation of Parameters of a Linear System with Gaussian Inputs, J. Electronics Control, (1963).

5. Groff, J. N., The Analyses of Stationary Stochastic Processes Using Spectral and Autoregression Techniques, AMSAA TR No. 63, Vol I., September 1972.

6. McRuer, D. T., etal, Human Pilot Dynamics in Compensatory Systems: Theory, Models, and Experiments with Controller-Element and Forcing Function Variations, AFFDL-TR-65-15, July 1965.

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APPENDIX A

DOCUMENTATION OF ESTIMATION ALGORITHMS

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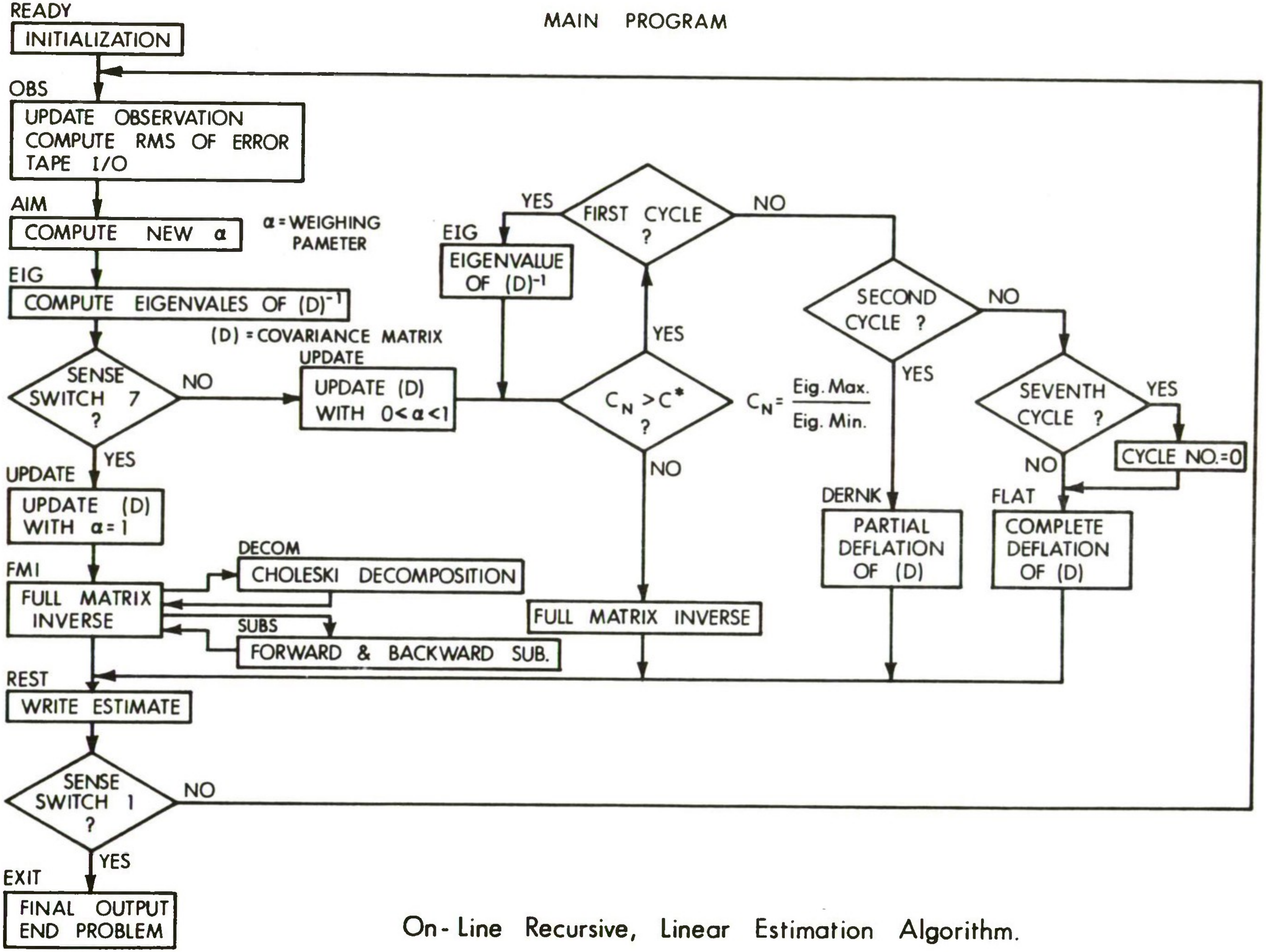
MAIN

PURPOSE: Overall Control of the Estimation Algorithm

SPECIFIC FUNCTIONS:

1) Call Ready
2) Set-up Estimation Cycle
3) Track Eigenvalues
4) Up-date Covariance Matrix, $D \equiv DEMAT^{-1}$
5) If SSW(7) then Full Inverse with no Weighting
6) If SSW (1) then Stop.

MAIN PROGRAM



On-Line Recursive, Linear Estimation Algorithm.

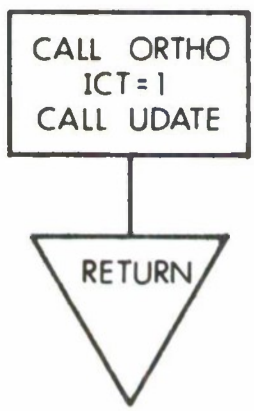
FLAT

PURPOSE: Controls Complete Deflation Unconditionally

SPECIFIC FUNCTIONS:
1) Call on Orthogonalization Routine
2) Set No. of Good Rows to 1
3) Calls UDATE for new B

INPUTS REQUIRED:
1) DEMAT
2) x, y
3) BI (i.e. previous model)

SUBROUTINE FLAT



SUBROUTINE EIG

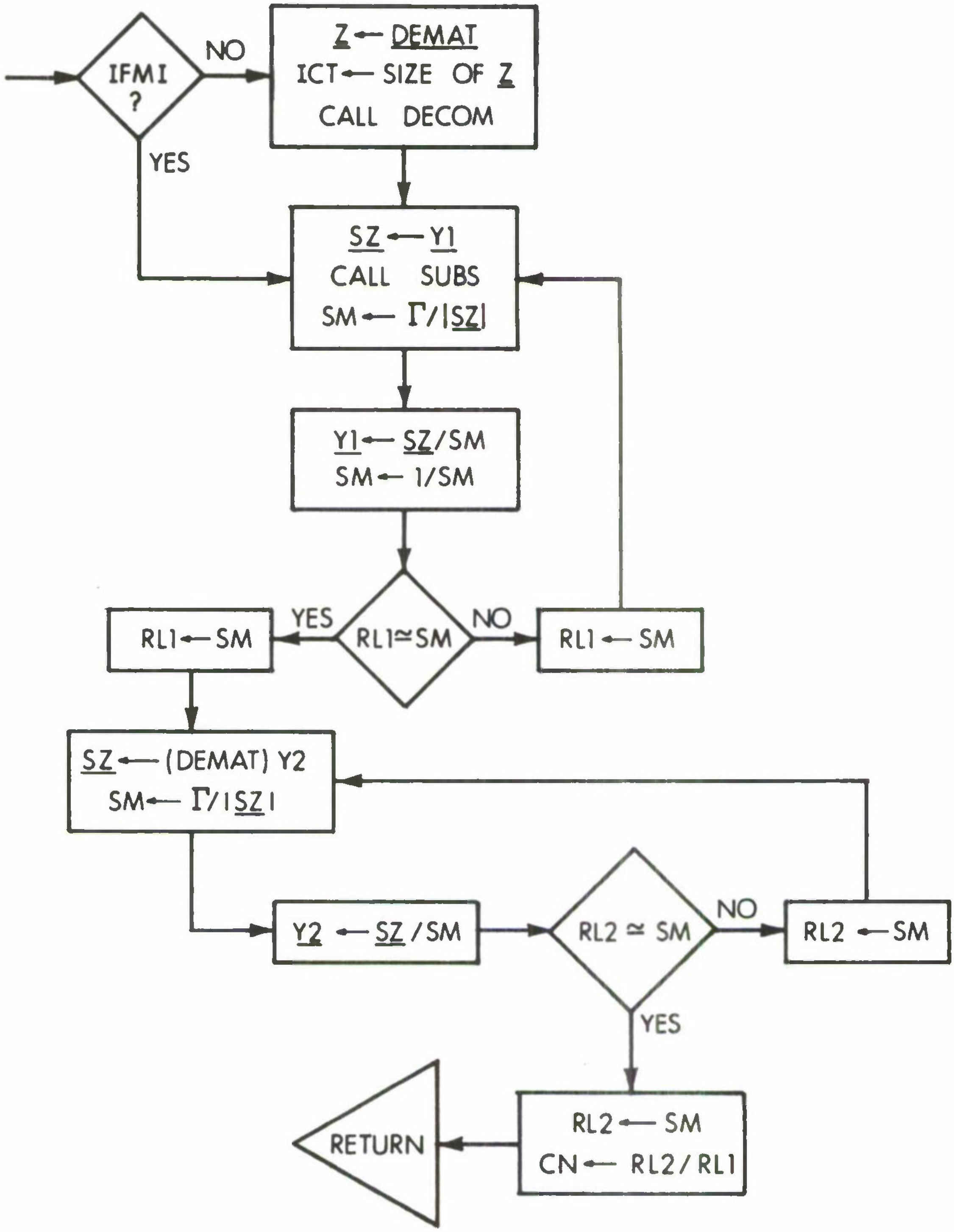
PURPOSE: To Approximate the Eigenvalues of the Inverse of the Covariance Matrix.

SPECIFIC FUNCTIONS:
1) Decompose DEMAT
2) Inverse Power Iteration for RL1
3) Power Iteration for RL2
4) $\Gamma / |\underline{SZ}|$: Largest absolute value of $\underline{SZ}$

INPUTS:
1) DEMAT

OUTPUTS:
RL1 - Smallest Eigenvalue of DEMAT
RL2 - Largest Eigenvalue of DEMAT

SUBROUTINE EIG



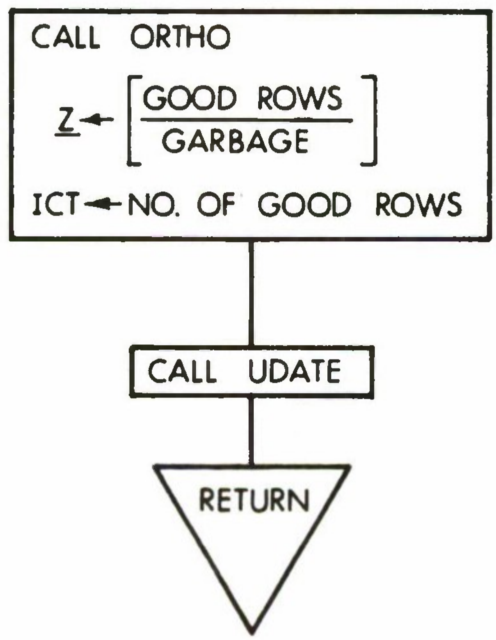
SUBROUTINE DERNK

PURPOSE: Controls Partial Deflation of DEMAT

SPECIFIC FUNCTIONS:
1) Calls Orthogonalization
2) Isolates Good Rows In Upper Part of $\underline{Z}$
3) Call UDATE for New $\underline{B}$

INPUTS:
1) DEMAT
2) $\underline{x}$, y
3) $\underline{BI}$ (i.e. previous model)

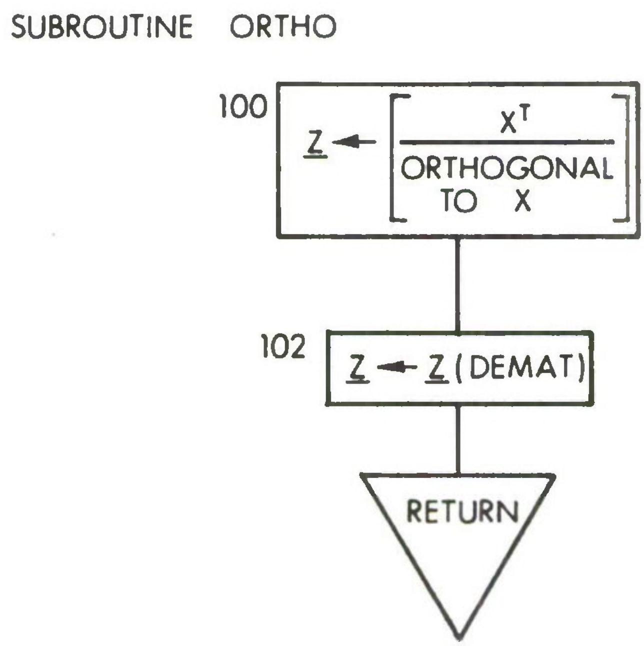
SUBROUTINE DERNK



SUBROUTINE ORTHO

PURPOSE: To multiply DEMAT by a matrix which has mutually orthogonal rows. The first row is x.

SPECIFIC FUNCTIONS: 1) Gram Schmidt Orthogonalization with respect to x
2) Multiply result of 1) by DEMAT.



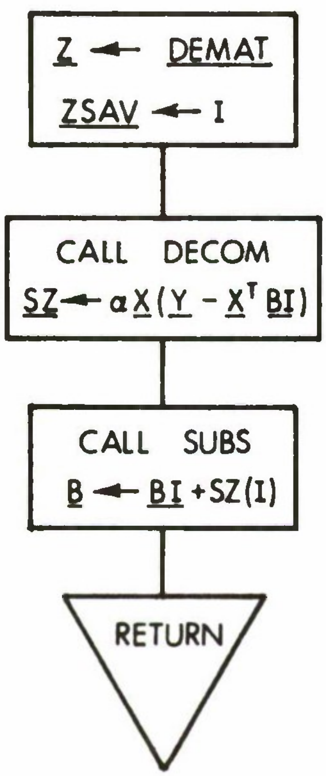
SUBROUTINE FMI

PURPOSE: Controls Full Inversion of DEMAT

SPECIFIC FUNCTIONS:
1) Decompose DEMAT
2) Compute Right Hand Side
3) Call SUBS
4) Compute New Model, $\underline{B}$

INPUTS:
1) DEMAT
2) $\underline{x}$, $\underline{y}$
3) $\underline{BI}$

SUBROUTINE FMI



SUBROUTINE DECOM

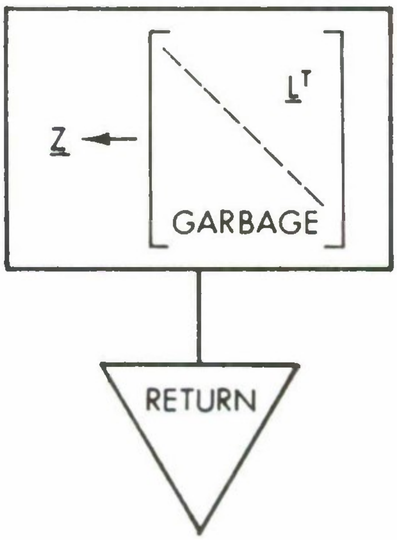
PURPOSE: Decompose a Symmetric Positive Definite Matrix Stored in $\underline{Z}$

SPECIFIC STEPS: Choleski Decomposition

INPUTS:
1) $\underline{Z}$
2) Rank of $\underline{Z}$ in ICT

OUTPUT:
1) L of $\underline{Z} = \underline{L}\ L^T$ stored in upper right triangular part of $\underline{Z}$

SUBROUTINE DECOM: ENTER WITH ICT ≡ NO. OF ROWS OF $\underline{Z}$ (i.e. ICT ≡ RANK OF $\underline{Z}$)



SUBROUTINE SUBS

PURPOSE: Forward and Back Substitution in Solution of

$\underline{L}\ \underline{Y} = \underline{B}^*$ By Forward Substitution

$\underline{L}^T\underline{X} = \underline{Y}$ By Back Substitution

SPECIFIC FUNCTIONS: 1) Forward Substitution

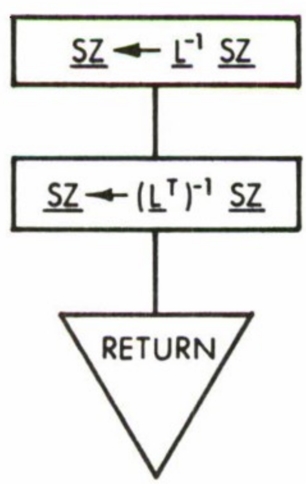
2) Back Substitution

INPUTS: 1) $\underline{L}$ in upper right part of $\underline{Z}$

2) Right hand side in $\underline{SZ}$

OUTPUTS: 1) Solution in $\underline{SZ}$

SUBROUTINE SUBS: $\underline{L}^T$ IS IN UPPER TRIANGLE OF $\underline{Z}$ WITH RANK OF ICT. $\underline{SZ}$ BEGINS AS RIGHT HAND SIDE.



SUBROUTINE UDATE

PURPOSE: Complete the Generalized Inversion of the Matrix, $\underline{Z}$, Having Known Rank, ICT.

SPECIFIC FUNCTIONS:

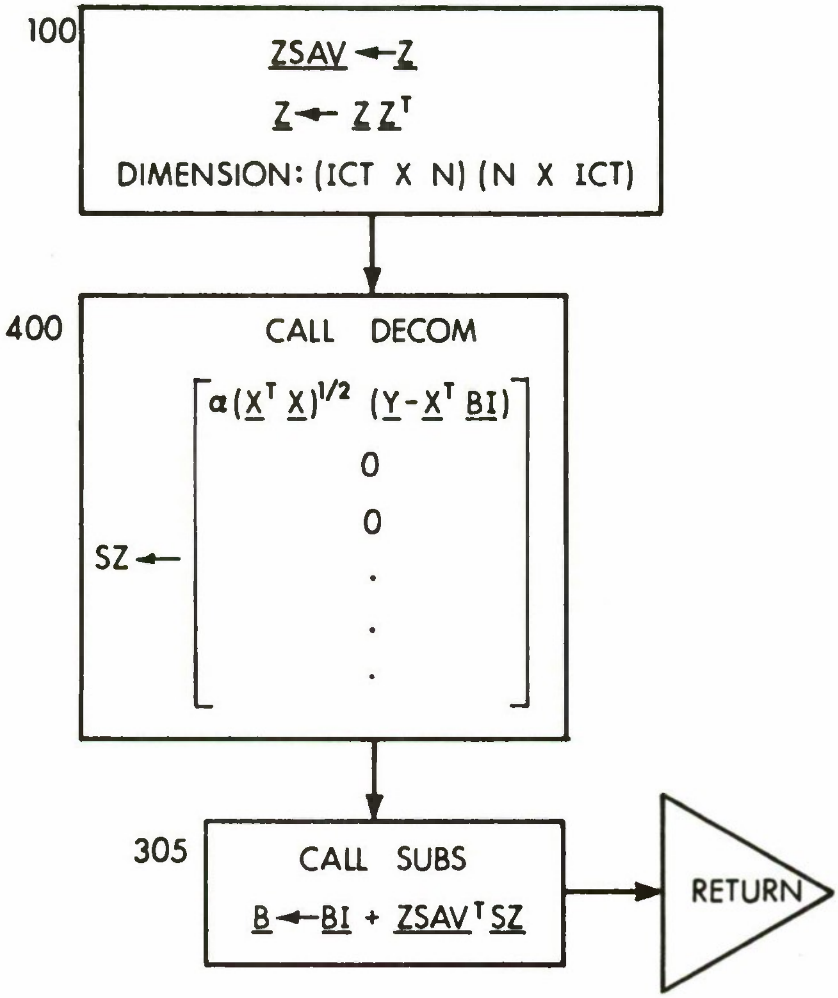
1) Save $\underline{Z}$ in $\underline{ZSAV}$
2) Form Symmetric Positive Definite Matrix, $\underline{Z} \leftarrow \underline{Z}\ \underline{Z}^T$
3) Solve $\underline{Z}\ \underline{X} = B^*$ for $\underline{X}$ by decomposing $\underline{Z}$, forming the right side, $\underline{B}^*$ and using SUBS.
4) Update the Model, $\underline{B}$
5) $\underline{B} \leftarrow \underline{BI} + \underline{ZSAV}^T \underline{Z}^{-1}\ \underline{B}^*$ (Pseudo Inverse)

INPUTS:

1) $\underline{Z}$, ICT (Matrix and Rank)
2) $\underline{x}$, y
3) $\underline{BI}$

NOTE: Any estimation cycle which uses UDATE must have already used ORTHO.

SUBROUTINE UDATE



SUBROUTINE AIM

PURPOSE: To Compute a New Weighting Parameter, α. Precess the Model.

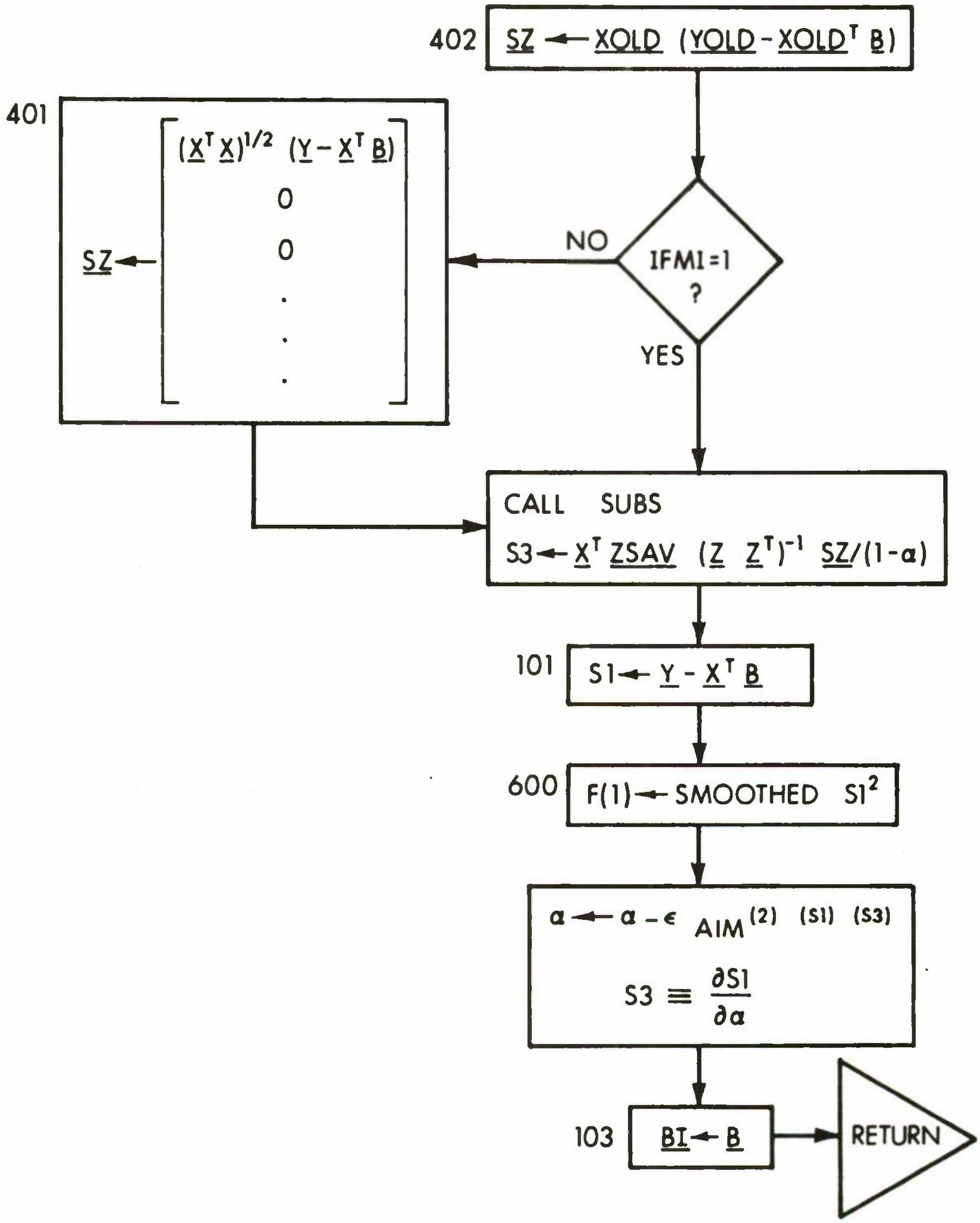
SPECIFIC FUNCTIONS:

1) Set up right hand side of $\underline{A}\,\frac{\partial \underline{B}_n}{\partial \alpha} = B^*$. Conditioned upon whether $\underline{A}$ is full rank or not.
2) Solve for $\frac{\partial B_n}{\partial \alpha}$ by inverting $\underline{A}$ using new right hand side.
3) Compute prediction error and smoothed prediction error.
4) Compute new α using steepest descent.
5) $\underline{BI} \leftarrow \underline{B}$; Precess the model index.

INPUT

1) $\underline{x}$, y
2) $\underline{XOLD}$, YOLD
3) Decomposed DEMAT and ZSAV
4) $\underline{B}$; previous model.
5) EPAIM, step size in steepest descent.

SUBROUTINE AIM



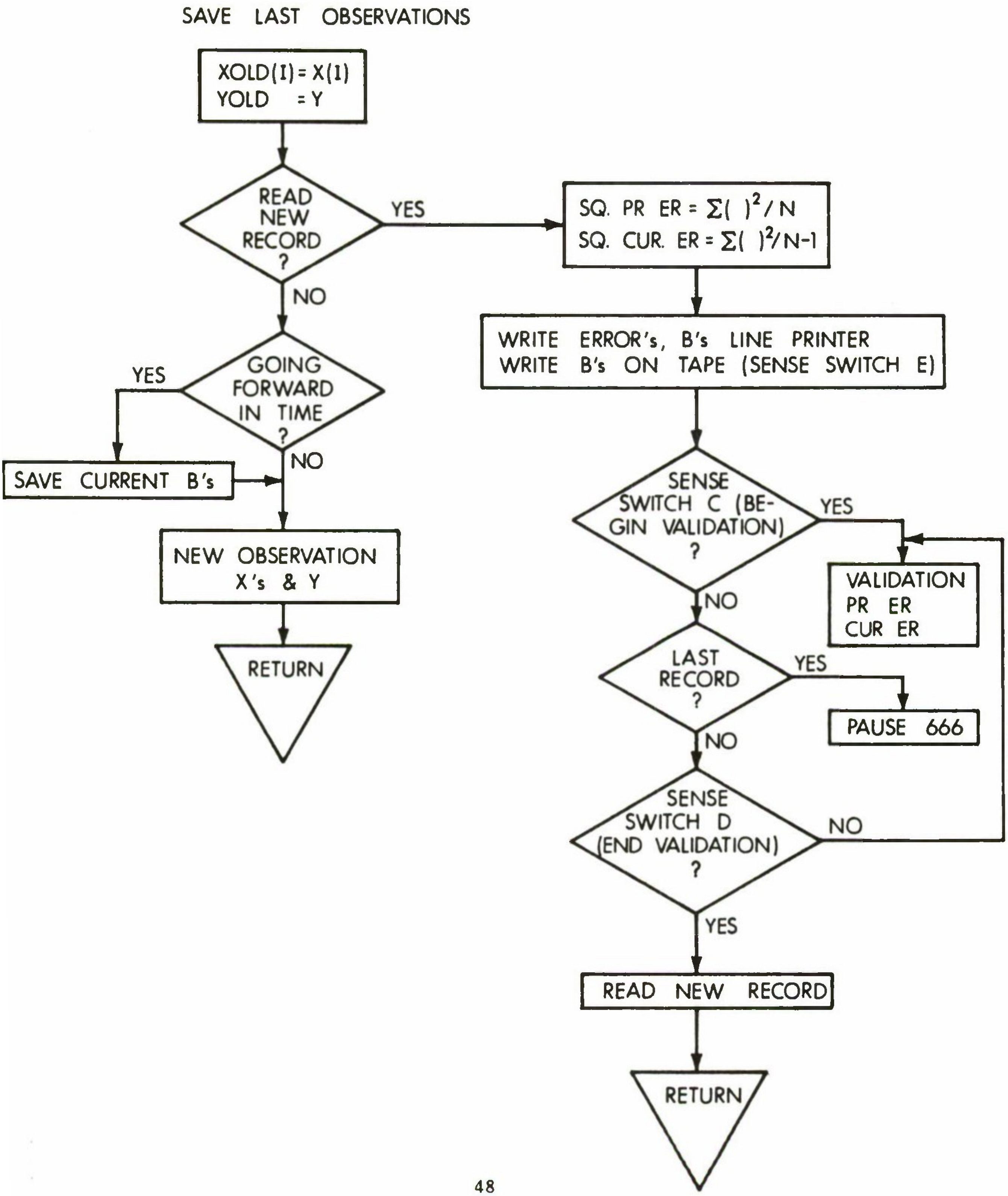
SUBROUTINE OBS

PURPOSE: Observe new data and precess last observation

SPECIFIC FUNCTIONS:

1) Save last observation
2) Save estimation coefficients (forward pass thru data).
3) Observe new data
4) Compute squared prediction error and squared current error for each tracking record.
5) Validation of estimation coefficients with other tracking records for any target type.

SUBROUTINE OBS



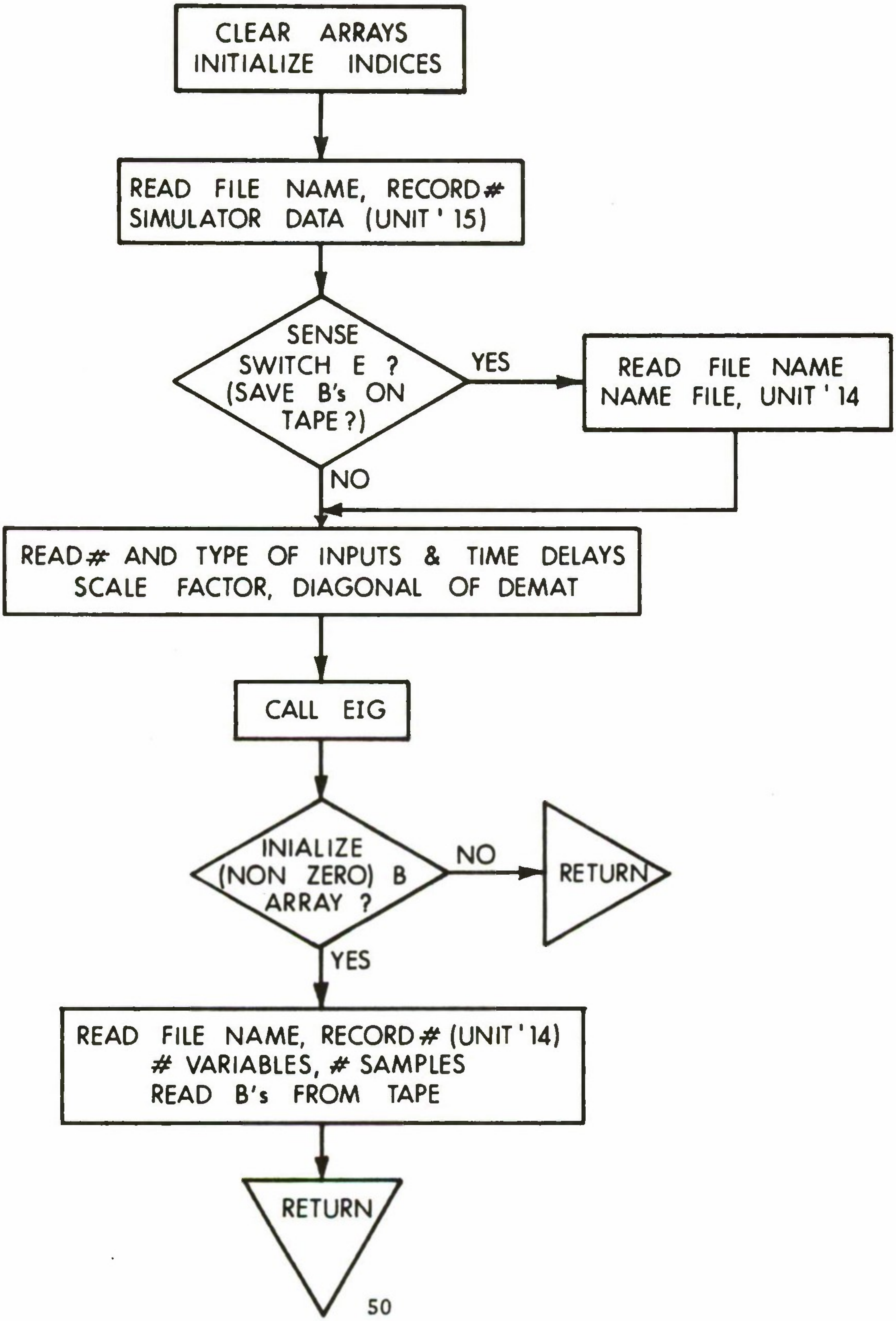
SUBROUTINE READY

PURPOSE: General Initialization

SPECIFIC FUNCTIONS:

1) Clear arrays and set indices.
2) Position input tape.
3) Name and position optional output tape.
4) Input model configuration
5) Initialize co-variance matrix diagonal values

SUBROUTINE READY



APPENDIX B

SYSTEM DESCRIPTION
AND
ANALYTICAL MODEL

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B.1 INTRODUCTION

The Vulcan Air Defense System (VADS) is an antiaircraft artillery (AAA) weapon that provides close-in defense against low-flying aircraft and can be used against ground vehicles such as trucks, personnel carriers, and other lightly armored vehicles. The VADS utilizes a 20-mm6 barrel Gatling type gun having a fire rate of 3000 rounds per minute with preselected burst lengths or 1000 rounds per minute with burst lengths under operator's trigger control. The VADS is a manually operated system equipped with a range-range rate only radar. The gunner is required to identify, acquire and track the target and operate the trigger for firing.

The fire control system uses a two axes lead computing sight, mounted on the gun trunnion, to generate an angular displacement between the gun tube and the gunner's line-of-sight. The gunner must acquire the target in the sight optics and maintain the target in the optical sight reticles (15m̸ and 60m̸ concentric circles) by controlling the rate of gun motion in azimuth and elevation. The gun rates in conjunction with the sight sensitivity parameter, T_N, which is approximately equal to ballistic time of flight of the round, are used by the lead computing sight to generate the lead angle between the gunners line-of-sight and gun position. The fire control prediction assumes the target is flying a straight line course at constant velocity.

The radar provides only the present range and range rate. The instantaneous range and range-rate data are major inputs to the current generator which provides T_N to the lead computing sight. The radar's position relative to the target depends upon the gunners ability to track the target. It is positioned in angle by the gun mount position in azimuth and by the radar's own servo loops, which receive input signals of gun elevation and two components of lead angle, elevation and traverse. The radar must stay within ± 4 degrees of the target to maintain lock-on.

B.2 ANALYTICAL MODELS

The mathematical representation of VADS is a nonlinear, time varying model of the Vulcan fire control system. The VADS model is comprised of submodels for the sight display, lead computing sight, sight current generator, turret servo and gun dynamics, hand controller and the operator. A model of the radar was developed but not included in the analytical model. Instead, a target model is used to provide the flight path parameter inputs to the analytical model. The block diagram of the VADS mathematical model is shown in Figure B-1.

B.2.1 Sight Display.

The sight display is actually an integral part of the lead computing sight, however, it is considered either as separate component or combined with the man model in modeling the VADS.

The sight display is the movable sight reticle. It consists of two concentric circles, 15 and 60₥ in diameter. The total field of view of the XM61 sight is approximately 400 mils and the diameter of the display aperature is 4 inches. Hence, the sight display is modeled as a constant, 10 inches/radian.

B.2.2 Hand Controller.

The operator's command to the system is transmitted through his manipulation of a hand controller (handlebars). The handlebars are rotated like steering an automobile for gun azimuth commands and are rotated vertically for elevation commands to the gun. The deflection of the handlebars causes a voltage to be applied to the inputs of the rate servo loop, thereby driving the turret and gun. The output voltage of the hand controller is a nonlinear function of handlebar position and rate. The differential equation relating the servo amplifier voltage, V, to handlebar deflection, δ_{HB}, is

$$V = K_\theta \, \delta_{HB} + K_L \, \dot{\delta}_{HB} \qquad \text{B.2.1}$$

where K_θ is the nonlinear function relating voltage output to displace-

ment of handlebars and K_L is the aided laying tachometer gain. This nonlinearity will be discussed in more detail in subsequent paragraphs. The K_L gain can be linearized to a constant without degrading the system performance. The aided laying tach's main function is to provide an output which is related to rate of change of handlebar deflection during the acquisition phase.

B.2.3 Turret Servo Loop.

The turret servo loops produce angular gun rates in azimuth and elevation in response to handlebar position, i.e. the turret subsystem is a tyle 1 servo system. The equations of motion for the turret servo loops are made up from the gun dynamics, operational amplifiers, torquer motors, tachometers and associated electronics. A simplified block diagram of the uncoupled servo loop for either axis is shown in Figure B.2. The gain, K_1, in the tach feedback is a nonlinear function of handlebar deflection.

Assuming that all the parameters are constants, a transfer function relating gun position, G, to voltage input, V, can easily be derived, i.e.

$$\frac{G}{V} = \frac{\dfrac{K_A}{N(K_E + K_A K_1 K_F)}}{S\left[\dfrac{R_m J_T t_m}{K_T(K_E + K_A K_1 K_F)} S^2 + \dfrac{R_m J_T}{K_T(K_E + K_A K_1 K_F)} S + 1\right]} \qquad \text{B.2.2}$$

where

K_A = Servo amplifier gain

K_T = Motor torque constant

K_E = Motor back emf constant

K_1 = Variable gain dependent upon handlebar deflection and tachometer feedback resistance.

K_1 = Tachometer gain

N = Gain ratio

R_m = Motor resistance

J_T = Combined motor inertia and load inertia referred to motor shaft

t_m = Electrical time constant of motor

Let
$$\tau_G = \frac{R_m J_T}{K_T(K_E + K_A K_1 K_F)} \qquad \text{B.2.3}$$

and
$$K_G = \frac{K_A}{N(K_E + K_A K_1 K_F)} \qquad \text{B.2.4}$$

then, Equation B.2.2 can be written as

$$\frac{G}{V} = \frac{K_G}{S(t_m \tau_G S^2 + \tau_G S + 1} \qquad \text{B.2.5}$$

The time constant of the motor, t_m, is 0.002 sec. Since t_m is small (i.e. $t_m \leq \leq 1$), Equation B.2.5 can be reduced and approximated by

$$\frac{G}{V} = \frac{K_G}{S(\tau_G S - 1)} \qquad \text{B.2.6}$$

This is the uncoupled model of the turret servo loop.

Equation B.2.3 and B.2.4 show that both τ_G and K_G depend on K_1 which is a function of handlebar position, δ_{HB}. The variable gain K_1 cannot readily be computed. In order to reduce modeling errors, τ_G and K_G are assumed to be constant and all nonlinearity between δ_{HB} and turret servo output are modeled in the hand controller's nonlinear gain K_θ. This nonlinearity is approximated from measurements taken from the VADS hybrid simulator.

In the development of the man model, the operator is assumed to be operating the system in a tracking only mode (nonfiring), and the dynamic coupling between the gun axes is assumed to be zero. Hence, Equation B.2.6 is used as the model of turret servo and gun dynamics.

B.2.4 Sight Current Generator.

For the purpose of this modeling effort, the sight current generator is defined as the subsystem which produces the sight sensitivity, T_N, for the lead computing sight. Neither a discussion on principles of operation nor any developed mathematical model will be presented in this document.[(1)]

Instead of using a analytical model of the sight current generator, a computer model is used to generate the theoretically correct T_N for a straight and level flight path. From the known velocity and position of the target and the ballistic data of the ammunition, an interactive process is used to calculate the future range at which a projectile would intercept the target. Using the computed values from the interactions, the theoretical T_N can be computed continuously using the following equations,[(2)]

$$T_N = \frac{t_f}{\frac{D_F}{D_P} + \sigma \left[\frac{D_F}{D_P} - \frac{D_P}{D_F} \left(\frac{1}{1 - V \frac{dt_f}{dD_F} \cos \alpha} \right) \right]} \qquad \text{B.2.7}$$

where t_f - projectile time-of-flight

σ - angle between target flight path and future range vector

D_P - present range

D_F - future range

V - velocity of target

For a known straight and level flight path, this equation provides values of T_N which are approximately correct.

B.2.5 Lead Computing Sight.

The lead computing sight is the "heart" of the Vulcan fire control system. It performs the lead computation required to point the gun barrels ahead of the present position of the target. Since the projectile have a finite time of flight, the sight predicts the future position of the target, where the gun tube must be aimed, in order for the projectile to hit the target.

Its principle of operation is based on the precessional law of a gimballed gyroscope. The case of the sight, which houses an electromagnet, is mounted so that it moves with the gun. An eddy current disk is attached to the rotor of the two axis gimballed gyroscope. The electromagnet is energized by a current signal generated in the sight current generator. When a moving target is tracked, eddy current forces are generated in the disk, creating a torque that causes the spin axis of the gyro to follow the gun with a displacement (lag) sufficient to provide the torque needed to precess the gyro at the gun rate. The magnitude of the electromagnet current determines the angular lag between the gun and the gyroscope spin axis for a given tracking rate. This angle is a measure of the lead angle being developed. Actually the reticle orientation relative to the gun differs from the gyro orientation relative to the gun by a constant ratio, σ. This permits instantaneous response between the gun motion and reticle motion. An additional quantity, referred to as superelevation, is generated in the elevation axis of the lead computing sight. Superelevation compensates for ballistic drop of the projectile.

A simplified model of the lead computing sight is developed by considering the motion of the sight, reticle and gun in a single plane. Assume the Vulcan is tracking a moving target in the horizontal plane as deplicted in Figure B.3. From this figure the following angular relations are obtained:

$$A_G - A_{Gyro} = \text{gyro lag} \qquad \text{B.2.8}$$

$$A_G - A_R = \lambda \qquad \text{B.2.9}$$

$$A_1 - A_R = \theta_\varepsilon$$

where A_G = gun line of sight

A_{Gyro} = gyro line of sight

A_R = reticle line of sight

A_T = target line of sight

λ = lead angle

θ_ε = track error

If the sight gyroscope with the eddy current disk mounted on its spin axis is rotating in an electromagnet field that is fixed to the gun, the precessional rate of the gyroscope is related to the displacement between the gun and the gyroscope by

$$\dot{A}_{Gyro} = PI^K \sin \left(\frac{A_G - A_{Gyro}}{K_S}\right) \qquad \text{B.2.11}$$

where P, I^K and K_S are related to parameters of the sight current generator and the lead computing sight. If $PI^K = \frac{1}{T_N}$ and $K_S = (1 + \sigma)$, the small angle approximation of Equation B.2.11 becomes

$$\dot{A}_{Gyro} = \frac{A_G - A_{Gyro}}{(1 + \sigma)T_N} \qquad \text{B.2.12}$$

The reticle is controlled by the sight gyroscope but their lines-of-sight relative to the gun differ by a fractional part of the lead angle, $\sigma\lambda$, i.e.

$$A_R - A_{Gyro} = \sigma\lambda \qquad \text{B.2.13}$$

Substituting Equation B.2.13 into Equation B.2.9, the angular relations between the gun and gyro is

$$A_G - A_{Gyro} = (1 + \sigma)\ \lambda \qquad \text{B.2.14}$$

The right side of this equation is the gyro lag relative to the gun. The time derivative of this equation is

$$\dot{A}_G - \dot{A}_{Gyro} = (1 + \sigma)\ \dot{\lambda} \qquad \text{B.2.15}$$

Solving Equation B.2.12, B.2.14 and B.2.15 in terms of λ and A_g, yields a differential equation for the lead angle

$$\dot{\lambda} = \frac{\dot{A}_G}{(1+\sigma)} - \frac{\lambda}{T_N} \qquad \text{B.2.16}$$

This equation is only valid for computing the lead angle in a single plane.

In reality, the Vulcan fire control system is solving for two components of lead angle, traverse and elevation. Since the gun, reticle and sight gyro are not in the same coordinate system, transformulations are required to project the respective vectors into the same coordinate system. Neglecting superelevation and using small angle approximations, the dynamic equations of the sight in traverse, λ_T, and elevation, λ_E, can be approximated by

$$\dot{\lambda}_T = \frac{\dot{A}_G \cos E_G}{1 + \sigma} - \frac{\lambda_T}{(1 + \sigma)T_N} \qquad \text{B.2.17}$$

$$\dot{\lambda}_E = \frac{\dot{E}_G}{(1 + \sigma)} - \frac{\lambda_E}{(1 + \sigma)T_N} \qquad \text{B.2.18}$$

where E_G is the gun position in elevation. Although, the term, cos E_G, is a liberal approximation of the required trnasformulations, the lead angles obtained from Equations B.2.17 and B.2.18 compare favorable with the results obtained from experiments using the Vulcan sight and more

elaborate analytical models that take into account the coordinate transformations.[1,3]

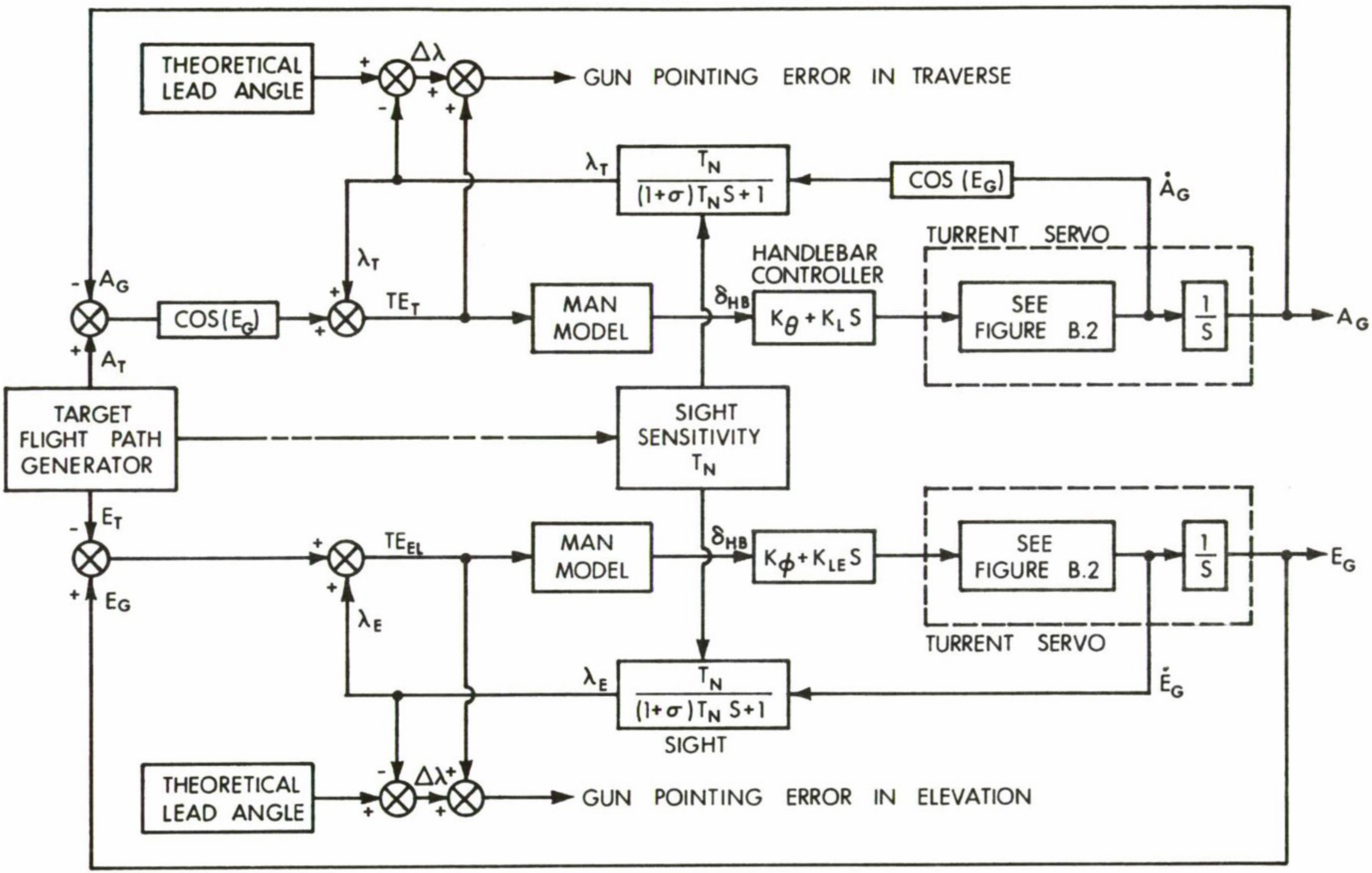

Figure B.1 Block Diagram of VADS Engineering Model.

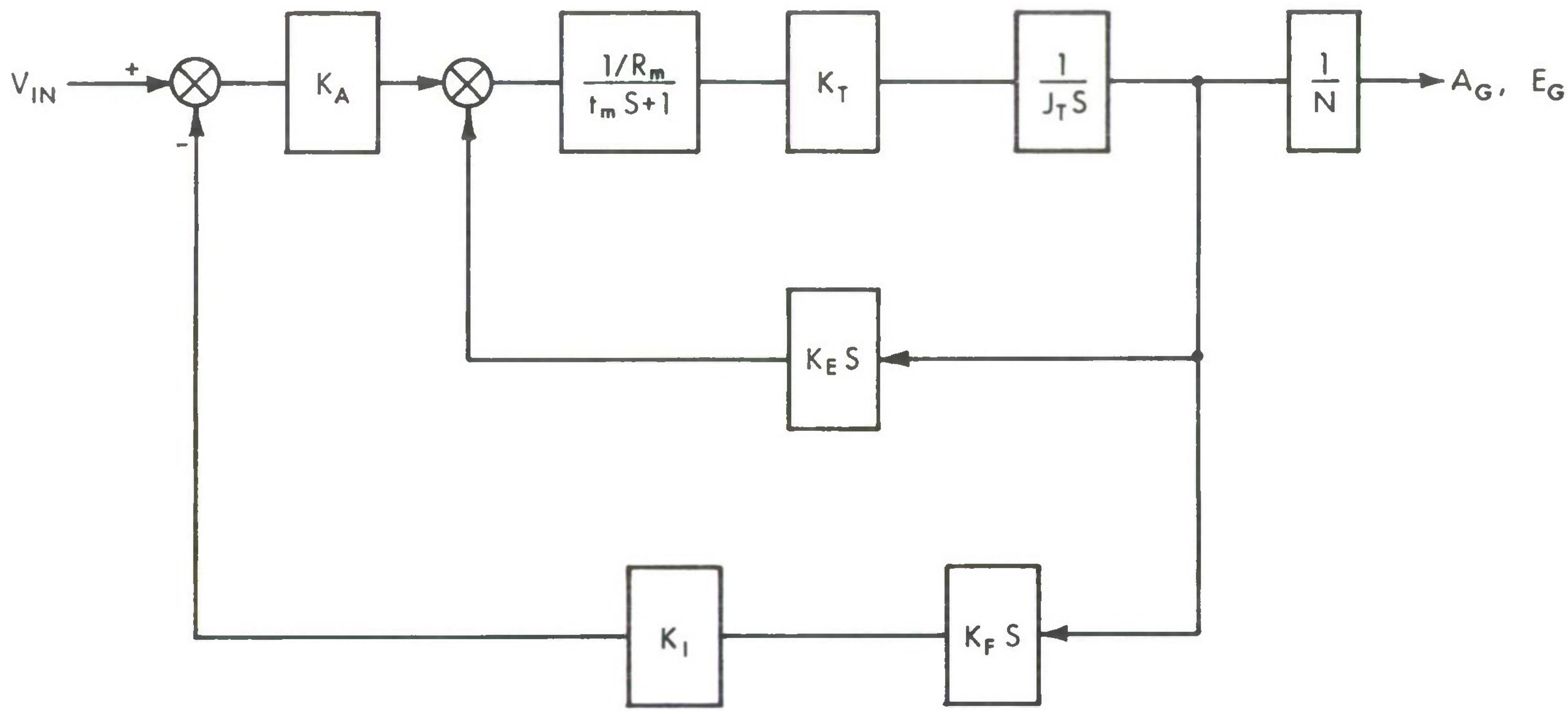

Figure B.2 Block Diagram of Uncoupled Turrent Servo for Either Axes.

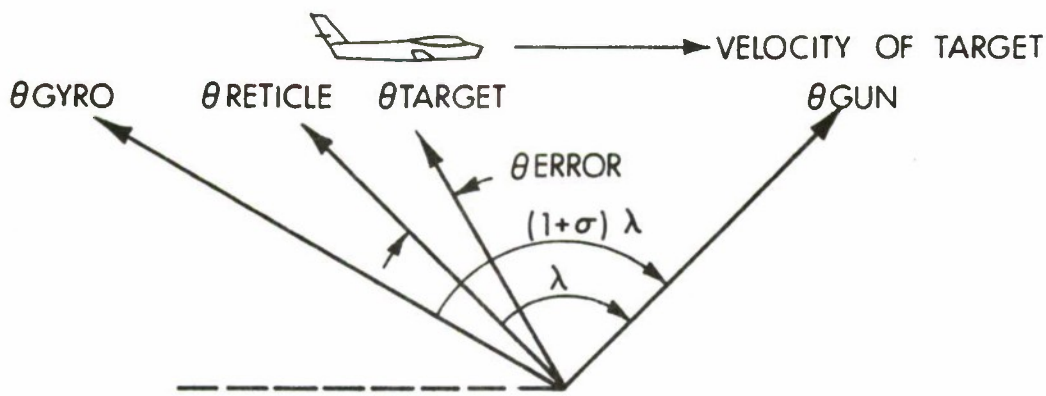

Figure B.3 Single Axis Lead Computing Sight Geometry.

REFERENCES

1. Unpublished Note on the Vulcan Sight Current Generator Model, Fire Control Division, Frankford Arsenal, Pennsylvania.

2. AMCP 700-399, Engineering Design Handbook, Fire Control Series, Section 3, October 1970.

3. General Electric, Fire Control for Vulcan Air Defense Systems XM163 and XM167, General Electric Report No. 66APB541-9, January 1968.

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APPENDIX C

TIME HISTORIES OF TRACKING ERROR FROM VADS SIMULATION AND DEVELOPED MAN MODEL

The figures in this appendix show the tracking responses of the actual man and the developed man model. The results are shown for four different target conditions. The target is flying at a constant velocity and altitude on a straight fly-by course.

The plots for the actual man are the averages of a trained gunner tracking the same target ten times.

The data were obtained from a man operated simulation of the Vulcan Air Defense System's (VADS) fire control system. The different target fly-bys were designed to provide the gunner with tracking tasks ranging from easy to difficult.

The tracking response of the model approaches the magnitude but not the frequency of the actual gunner tracking.

One of man's limitations that tends to degrade system performance is his randomness. This effect is not included in the developed man model. The recursive estimated algorithm smoothed out this randomness in estimated the parameters of the model.

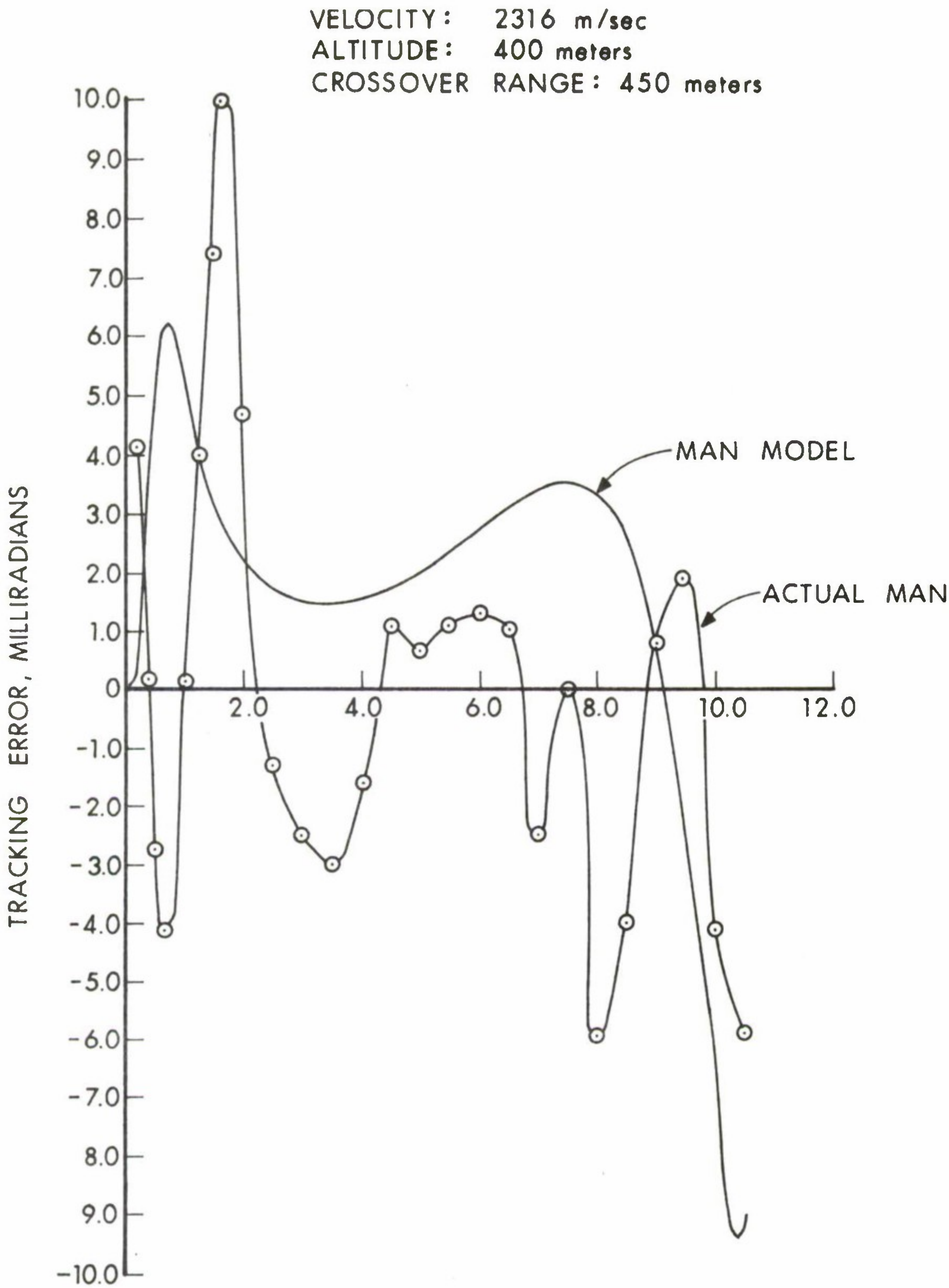

Figure C.1 Tracking Errors for Pass No. MMP1.

TARGET PARAMETERS

VELOCITY: 231.6 m/sec
ALTITUDE: 800 meters
CROSSOVER RANGE: 450 meters

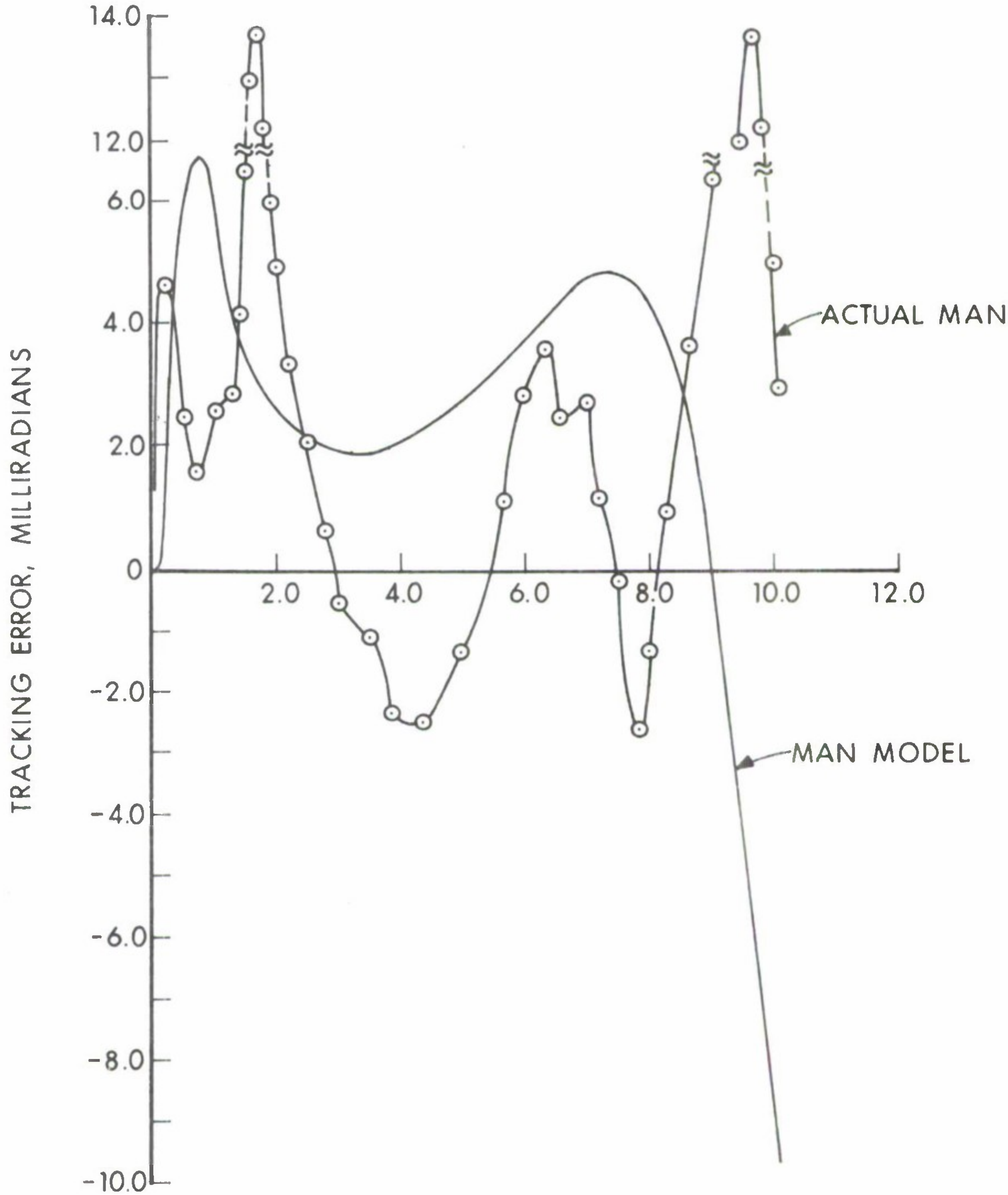

Figure C.2 Tracking Errors for Pass No. MMP3.

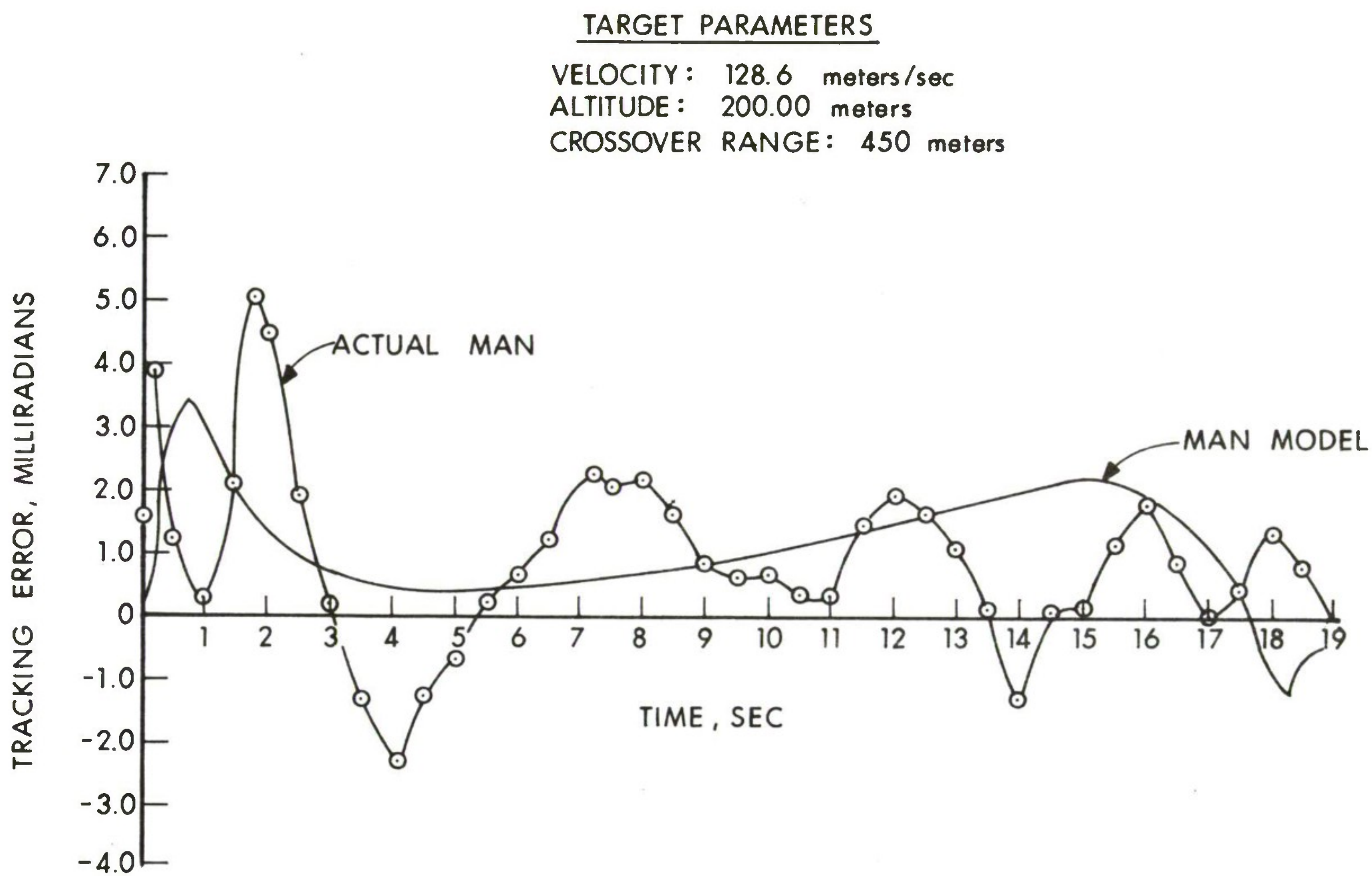

Figure C.3 Tracking Errors for Pass No MMP5.

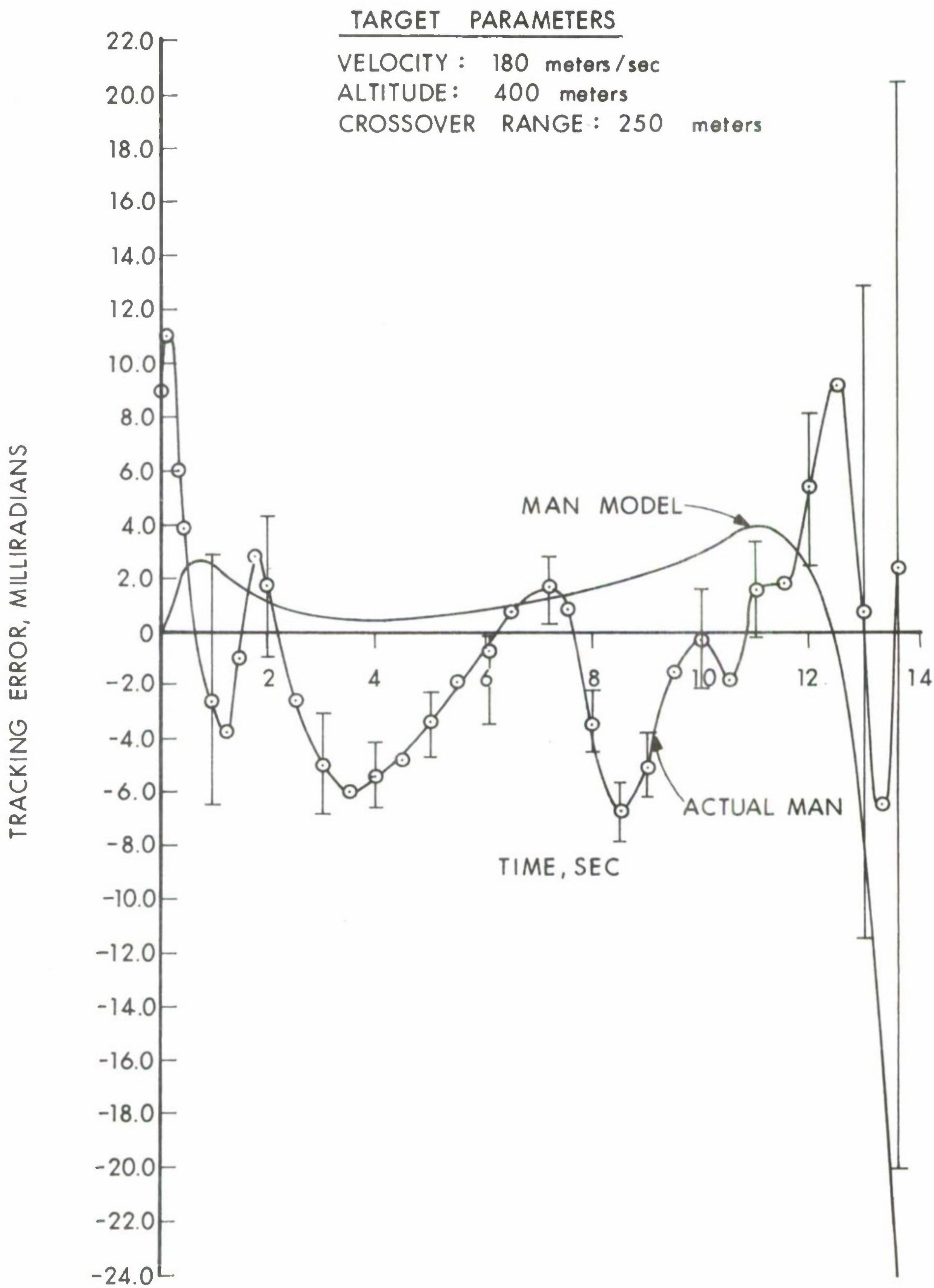

Figure C.4 Tracking Error for Pass No. MMP6.

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This report deals with the identification of unknown parameters in dynamic systems. In modeling a physical system, the problem of identifying the dynamics of a system are often encountered. The developed algorithm provides a tool to model all or parts of a dynamic system using input-output data sets from a real system. The methodology and techniques of this algorithm are based upon linear recursive estimation theory. The theoretical foundation and the pragmatics of utilizing the ensemble data to estimate the unknown parameters are discussed at length in the development of the algorithm. (cont'd)

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Item #20.
As an application of the algorithm, experimental data from a man-in-the-loop simulation is used to estimate the parameters of a single axis model of the gunner. The tracking response of the gunner model compare favorably with data obtained from the simulation. The differences in tracking responses are attributed to not including human randomness in the model.